STRENGTH OF MATERIALS II

#

Unit - 1
SPRINGS

Spring:
→ A Spring is a device, in which the material is arranged in such a way that it can undergo a considerable change, without getting permanently distorted.
→ A spring is used to absorb energy due to resilience, which may be restored as and when required.
→ The quality of a spring is judged from the energy it can absorb.

Stiffness of a spring:
The load required to produce a unit deflection in a spring is called spring stiffness or stiffness of a spring.

Types of Springs:

(i) Bending spring
(ii) Torsion Spring

Bending spring
A spring, which is subjected to bending only and the resilience is also due to it,

is known as bending spring.
Laminated springs or Leaf spring are also called bending springs.

Torsion spring:

A spring, which is subjected to torsion or twisting moment only and the resilience is also due to it, is known as a torsion spring.

Helical springs are also called torsion springs.

Some springs are subjected to bending as well as Torsion.

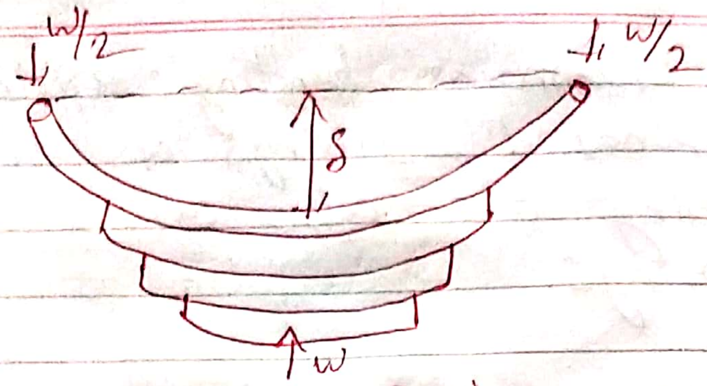
Forms of Springs:

The following types of springs are commonly used in Engineering practice.

1) Carriage springs or leaf springs
2) Helical springs.

Carriage springs:

The carriage springs are widely used in railway wagons, Coaches and road vehicles. These are used to absorb shocks.



Carriage Spring.

Helical Springs

→ It is a torsion springs and made up of a wire coiled into a helix.

→ The following are the important types of Helical Spring. They are.

1) Closely - coiled helical springs

2) Open - coiled helical springs.

Closely - coiled Helical Springs

→ In a closely coiled helical spring, the spring wire is coiled so close that the each turn is practically a plane at right angles to the axis of the helix and the wire is subjected to torsion.

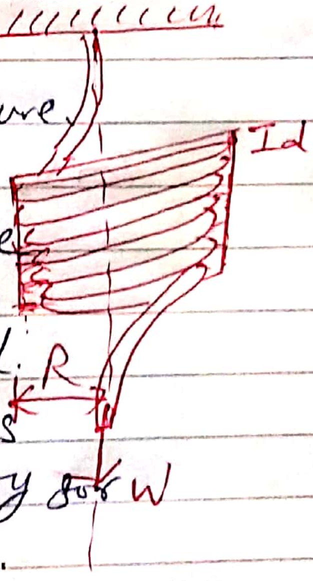
→ The bending stress is negligible as compared to the torsional stress.

→ A closely coiled helical spring may be subjected to

1) Axial loading
2) Angle twist

Closely - Coiled Helical Spring Subjected to an Axial load

Consider a closely - coiled helical spring subjected to an axial load as shown in figure.



Let

d = Diameter of the spring wire

R = Mean radius of the spring coil.

n = No. of turns of coils

C = Modulus of rigidity for the spring material.

W = Axial load on the spring

τ = Maximum shear stress induced in the wire due to twisting.

θ = Angle of twist in the spring wire.

δ = Deflection of the spring, as a result of axial load.

Load W will cause a twisting moment,

$$T = W \cdot R$$

Twisting moment

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$W \cdot R = \frac{\pi}{16} \times \tau \times d^3$$

Length of the wire

l = Length of one coil × No. of coils

$= 2\pi R \cdot n$

Torsion of circular shaft

$$\frac{T}{J} = \frac{C \cdot \theta}{l}$$

$$\theta = \frac{T \cdot l}{J \cdot C}$$

$J = \frac{\pi}{32} D^4$
(Polar moment of inertia)

$$= \frac{W R \cdot 2\pi R n}{\frac{\pi}{32} \times d^4 \times C}$$

$$\boxed{\theta = \frac{64 W R^2 n}{C d^4}}$$

Deflection of the spring,

$$\delta = R \cdot \theta$$

$$= R \times \frac{64 W R^2 n}{C d^4}$$

$$\boxed{\delta = \frac{64 W R^3 n}{C d^4}}$$

Energy stored in the spring

$$\boxed{U = \frac{1}{2} W \cdot \delta}$$

Stiffness of the spring.

$$S = \frac{W}{\delta}$$

$$\boxed{S = \frac{C d^4}{64 R^3 n}}$$

Problem.

A close-coiled helical spring is required to carry a load of 150N. If the mean coil diameter is to be 8 times of the wire. Calculate these diameters. Take maximum shear stress as 100MPa

Solution:

Given, Load (W) = 150N

Diameter of coil (D) = 8d

(Where d is the diameter of the wire).

(or)

Radius = 8d/2 = 4d

Maximum shear stress (τ) = 100Mpa

= 100 N/mm²

We know that relation for the twisting moment

$$WR = \frac{\pi}{16} \times \tau \times d^3$$

$$150 \times 4d = \frac{\pi}{16} \times 100 \times d^3$$

$$d^2 = \frac{150 \times 4 \times 16}{\pi \times 100}$$

$$d^2 = 30.6$$

$$\boxed{d = 5.53 \text{ say } 6\,mm}$$

$$\boxed{D = 8d = 48\,mm}$$

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Problem:

A closely coiled helical spring of round Steel wire 5mm in diameter having 12 complete coils of 50mm mean ~~radius~~ diameter is subjected to an axial load of 100 N. Find the deflection of the spring and the maximum Shearing Stress in the material. Modulus of Rigidity (C) = 80 GPa

Solution:

Diameter of spring wire (d) = 5mm
No. of coils (n) = 12
Diameter of Spring (D) = 50mm
Radius of Spring (R) = 25 mm
Axial Load (W) = 100N
Modulus of Rigidity (C) = 80 GPa
$= 8.0 \times 10^3 \ N/mm^2$.

Deflection of the spring

$$\delta = \frac{64 W R^3 n}{C d^4}$$

$$= \frac{64 \times 100 \times (25)^3 \times 12}{(80 \times 10^3) \times (5)^4}$$

$$\boxed{\delta = 24 \text{ mm}}$$

Maximum Shearing Stress in the Material.

τ = Maximum shearing stress

we know that relation for the torque,

$$WR = \frac{\pi}{16} \times \tau \times d^3$$

$$100 \times 25 = \frac{\pi}{16} \times \tau \times (5)^3$$

$$2500 = 25.54\,\tau$$

$$\tau = \frac{2500}{25.54}$$

$$\boxed{\tau = 101.9\ N/mm^2}$$

3. A Closely coiled helical spring is to carry a load of 500N. Its mean coil diameter is to be 10 times of the wire diameter. Calculate these diameters if the maximum shear stress in the material of the spring is to be $80\ N/mm^2$.

Given.

Load $W = 500N$, $\tau = 80\ N/mm^2$

d = Diameter of wire, $D = 10d$

$R = 5d$

$$WR = \frac{\pi}{16} \times \tau \times d^3$$

$$500 \times 5d = \frac{\pi}{16} \times 80 \times d^3$$

$$d^2 = 159.25$$

$$\boxed{\begin{aligned} d &= 12.6\,mm \\ D &= 10 \times d = 126\,mm \end{aligned}}$$

4. In the problem 3, if the Stiffness of the spring is 20N per mm deflection and modulus of rigidity = 8.4×10^4 N/mm², find the number of coils in the closed coiled helical spring.

Given:

Stiffness, $S = 20\,N/mm$.

Modulus of rigidity, $C = 8.4 \times 10^4\,N/mm^2$

$W = 500\,N$, $\tau = 80\,N/mm^2$

$d = 12.6\,mm$, $D = 126\,mm$

$R = D/2 = 126/2 = 63\,mm$.

n = number of coils in the spring

Deflection,

$$\delta = \frac{64 W R^3 n}{C d^4} \rightarrow (i)$$

$$\therefore \quad s = \frac{Load}{\delta}$$

$$\delta = \frac{Load}{s}$$

$$\delta = \frac{500}{20} = 25\,mm$$

$$\boxed{\delta = 25\,mm}$$

Sub $\delta = 25\,mm$ in equ (1)

$$\delta = \frac{64 W R^3 \cdot n}{C d^4}$$

$$25 = \frac{64 \times 500 \times (63)^3 \times n}{8.4 \times 10^4 \times 12.6^4}$$

$n = 6.6$ say 7.0

$$\boxed{n = 7}$$

5) A closely coiled helical spring of round steel wire 10mm in diameter having 10 complete turns with a mean diameter of 12 cm is subjected to an axial load of 200N. Determine.

(i) the deflection of the spring

(ii) maximum shear stress in the wire

(iii) stiffness of the spring

(iv) Energy stored in spring.

Given;

Diameter of wire. $d = 10mm$

No. of turns, $n = 10$

mean diameter of coil $D = 12\,cm = 120mm$

Radius of coil $R = D/2 = 60\,mm$

Axial load, $W = 200N$

modulus of rigidity, $C = 8 \times 10^4\ N/mm^2$

(i) Deflection

$$\delta = \frac{64 W R^3 \times n}{C d^4}$$

$$= \frac{64 \times 200 \times 60^3 \times 10}{8 \times 10^4 \times 10^4}$$

$$\boxed{\delta = 34.5\ mm}$$

(ii) Maximum Shear Stress

$$\tau = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 200 \times 60}{\pi \times 10^3}$$

$$\boxed{\tau = 61.1 \, N/mm^2}$$

(iii) Stiffness of the Spring

$$s = \frac{W}{\delta}$$

$$= \frac{200}{34.5}$$

$$\boxed{s = 5.8 \, N/mm}$$

(iv) Energy stored in the Spring.

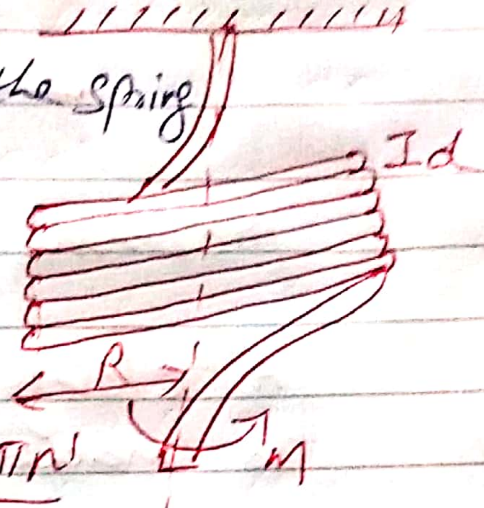
$$= \frac{1}{2} W \times \delta$$

$$= \frac{1}{2} \times 200 \times 34.5$$

$$= 3450 \, N.mm.$$

Closely Coiled Helical Spring Subjected to an Axial Twist

M : Moment or axial twist applied on the spring



$l = 2\pi R n = 2\pi R' n'$

$\frac{1}{R} = \frac{2\pi n}{l}$ and $\frac{1}{R'} = \frac{2\pi n'}{l}$

$$\frac{M}{I} = E \times \text{change of curvature}$$

$$= E\left(\frac{1}{R'} - \frac{1}{R}\right)$$

$$= E\left(\frac{2\pi n'}{l} - \frac{2\pi n}{l}\right)$$

$$= \frac{2\pi E}{l}(n' - n)$$

$$2\pi(n' - n) = \frac{Ml}{EI} \longrightarrow (i)$$

Total angle of bend

$$\phi = 2\pi(n' - n)$$

Sub $2\pi(n' - n)$ from eqn (i)

$$\phi = \frac{Ml}{EI}$$

Energy Stored in the Sping

$$\boxed{U = \frac{1}{2} M \cdot \phi}$$

problem:

A closely coiled helical spring is made of 10 mm diameter steel wire having 10 coils with 80 mm mean diameter. If the spring is subjected to an axial twist of 10 kN·mm. Determine the bending stress and increase in the number of turns. Take E as 200 GPa.

Solution:

Given: Diameter of spring wire (d) = 10 mm

No. of coils (n) = 10 ; Diameter of Coil (D) = 80 mm or R = 40 mm.

~~Ang~~ Axial twist (M) = 10 kN·mm
= 10×10^3 N.mm

E = 200 GPa = 200×10^3 N/mm².

Bending stress

$$\sigma = \frac{M}{I} \times y$$

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$$= \frac{10 \times 10^3}{I} \times 5$$

$$= 101.9 \text{ N/mm}^2$$

$$I = \frac{\pi}{64} \times d^4$$

$$= \frac{\pi}{64} \times (10)^4$$

$$= 490.9 \text{ mm}^4$$

Increase in the number of turns.

Length of coil, $l = 2\pi R n$
$= 2\pi \times 40 \times 10$
$= 800\pi$ mm

and Increase in the no. of turns.

$$n' - n = \frac{Ml}{EI} \times \frac{1}{2\pi}$$

$$= \frac{(10 \times 10^3) \times 800\pi}{(200 \times 10^3) \times 490.9} \times \frac{1}{2\pi}$$

$$\boxed{n' - n = 0.04}$$

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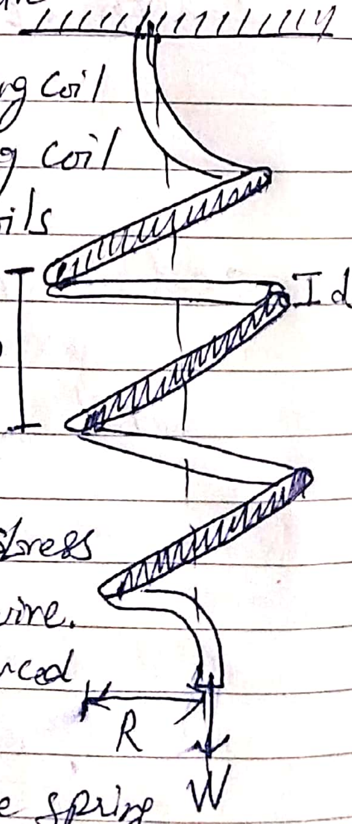
Open-Coiled Helical Springs

→ The spring wire is coiled in such a way, that there is largo gap between the two consecutive turns. As a result of this, the spring can take compressive load.

→ As like closed helical spring, open helical spring, may be subjected to
1) Axial loading
2) Axial Twist.

d = Diameter of the spring wire
R = Mean radius of the spring coil
P = Pitch of the spring coil
n = No. of turns of coils
C = Modulus of rigidity for the spring materials
W = Axial load on the spring
τ = Maximum shear stress induced in the spring wire.
σ_b = Bending stress induced in the spring wire.
δ = Deflection of the spring.
α = Angle of helix



The load W will cause a moment WR. This moment may be resolved into the following two components,

$$T = WR\cos\alpha \quad \text{(Twisting of coil)}$$
$$M = WR\sin\alpha \quad \text{(Bending of coil)}$$

Length of the spring wire.

$$l = 2\pi n R \sec\alpha$$

Twisting Moment

$$W \cdot R\cos\alpha = \frac{\pi}{16} \times \tau \times d^3$$

Bending stress

$$\sigma_b = \frac{M}{I} \times y$$

$$= \frac{WR\sin\alpha \cdot \frac{d}{2}}{\frac{\pi}{64} \times d^4}$$

$$\boxed{\sigma_b = \frac{32 WR\sin\alpha}{\pi d^3}}$$

Angle of twist

$$\theta = \frac{Tl}{JC}$$

$$\boxed{\theta = \frac{WR\cos\alpha . l}{JC}}$$

Angle of Bend due to bending moment

$$\phi = \frac{Ml}{EI}$$

$$\boxed{\phi = \frac{WR\sin\alpha . l}{EI}}$$

Word done by the load in deflecting the spring, is equal to the stress energy of the spring.

$$\frac{1}{2} W . \delta = \frac{1}{2} T . \theta + \frac{1}{2} M . \phi$$

$$W . \delta = T . \theta + M . \phi$$

$$= \left[WR\cos\alpha \times \frac{WR\cos\alpha . l}{JC} \right] + \left[WR\sin\alpha \times \frac{WR\sin\alpha . l}{EI} \right]$$

$$\delta = WR^2 l \left[\frac{\cos^2\alpha}{JC} + \frac{\sin^2\alpha}{EI} \right]$$

Now substituting $l = 2\pi n R$, $J = \frac{\pi}{32} d^4$ and $I = \frac{\pi}{64} d4$

$$\therefore \boxed{\delta = \frac{64 WR^3 n \sec\alpha}{d^4} \left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E} \right]}$$

If we substitute $\alpha = 0$ in the above equation, it gives deflection of a closed coiled spring, i.e.,

$$\delta = \frac{64 WR^2 n}{Cd^4}$$

Problem

An open coil helical spring made up of 10mm diameter wire and of mean diameter of 100mm has 12 coils, angle of helix being 15°. Determine the axial deflection and the intensities of bending and shear stresses under an axial load of 500N. Take C as 80 GPa and E = 200GPa.

Solution:

Given:

Diameter of wire (d) = 10mm

Mean diameter of spring (D) = 100 mm

Radius of spring (R) = 50 mm

No. of coils (n) = 12

Angle of helix (α) = 15°

Load (W) = 500N.

Modulus of Rigidity (C) = 80 GPa
$= 80\times10^3$ N/mm²

Modulus of Elasticity (E) = 200 GPa
$= 200\times10^3$ N/mm²

Deflection of the Spring

$$\delta = \frac{64 W R^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E}\right]$$

$$= \frac{64\times500\times50^3\times12\sec15°}{10^4}\left[\frac{\cos^2 15°}{80\times10^3} + \frac{2\sin^2 15°}{200\times10^3}\right]$$

$= 61.3$ mm.

Bending Stress in the Section.

$$\boxed{\sigma_b = \frac{M}{I}\times y}$$

Bending moment in the Coil,

$M = WR\sin\alpha$

$= 500\times50\sin15°$

$= 6470$ N.mm

Moment of Inertia of spring wire

$$I = \frac{\pi}{64} \times d^4$$

$$= \frac{\pi}{64} \times 10^4$$

$$I = 490.9 \, mm^4$$

$\therefore$ Bending Stress

$$\sigma_b = \frac{M}{I} \times y$$

$$= \frac{6470}{490.9} \times 5$$

$$\boxed{\sigma_b = 65.9 \text{ Mpa}}$$

Shear Stress induced in the wire

τ = Shear stress induced in the wire in N/mm^2.

$$W \cdot R \cos\alpha = \frac{\pi}{16} \times \tau \times d^3$$

$$500 \times 50 \cos 15^\circ \times \cos 15^\circ = \frac{\pi}{16} \times \tau \times (10)^3$$

$$\boxed{\tau = 123 \, N/mm^2}$$

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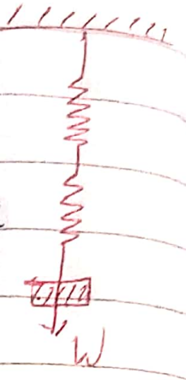
Springs in Series and Parallel

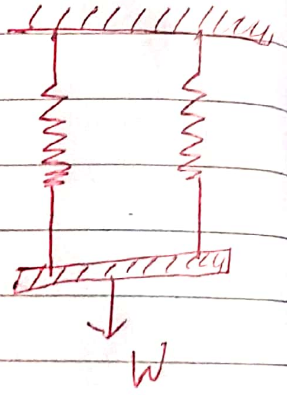
* Springs in Series



→ The total extension of the spring is equal to the algebraic sum of the extensions of the two springs.

→ Each spring subjected to same load applied at end of one spring

* Springs in parallel



→ The extension of each spring is the same

→ Load applied on the spring is shared by two springs.

problem

Two - close coiled helical springs wound from the same wire, but with different core radii having equal no. of coils are compressed between rigid plates at their ends. Calculate the maximum shear stress induced in each spring, if the ~~spring~~ wire diameter is 10mm and the load applied between the rigid plates is 500N. The core radii of the springs 100mm and 75 mm respectively.

Solution:

Given:

No. of coils in the outer spring (n_1) = n_2

(where n_2 is the no. of coils in the inner spring):

Diameter of the spring wire (d) = 10mm.

Load (W) = 500N.

Radius of outer spring (R_1) = 100mm

Radius of Inner spring (R_2) = 75mm

τ_1 & τ_2 = Shear stress developed in the outer & Inner spring.

W_1 & W_2 = Load shared by outer & Inner spring.

∴ ~~Deflection of~~

Relation equation.

Outer spring

$$W_1 R_1 = \frac{\pi}{64} \tau_1 d^3 \rightarrow (1)$$

Inner spring

$$W_2 R_2 = \frac{\pi}{64} \tau_2 d^3 \rightarrow (2)$$

The springs are held between two rigid plates, therefore deflections in both the springs must be equal.

$$\delta_1 = \delta_2$$

$$\delta_1 = \frac{64 W_1 R_1^3 n_1}{Cd^4}$$

$$= \frac{64 \times W_1 \times (100)^3 \times n_1}{C \times (10)^4}$$

$$= \frac{6400\, W_1 n_1}{C}$$

$$\delta_2 = \frac{64 W_2 R_2^3 n_2}{Cd^4}$$

$$= \frac{64 \times W_2 \times (75)^3 \times n_2}{C \times (10)^4}$$

$$= \frac{2700\, W_2 n_2}{C}$$

$\therefore \delta_1 = \delta_2$

$$\frac{6400 W_1 n_1}{C} = \frac{2700 W_2 n_2}{C}$$

$[n_1 = n_2]$

$$6400 W_1 = 2700 W_2$$

$$W_1 = \frac{27 W_2}{64}$$

we also know that

$$W_1 + W_2 = 500$$

$$\frac{27 W_2}{64} + W_2 = 500$$

$$W_2 = \frac{500 \times 64}{91}$$

$$\boxed{W_2 = 351.6\,N}$$

$$\boxed{\begin{array}{l} W_1 = 500 - W_2 \\ W_1 = 148.4\,N \end{array}}$$

sub above values in ①

$$① \Rightarrow W_1 R_1 = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$148.4 \times 100 = \frac{\pi}{16} \times \tau_1 \times 10^3$$

$$\boxed{\tau_1 = 75.6\,N/mm^2}$$

② ⇒

$$W_2 R_2 = \frac{\pi}{16} \times \tau_2 \times d^3$$

$$\tau_2 = \frac{351.6 \times 75 \times 16}{\pi \times 10^3}$$

$$\boxed{\tau_2 = 134.3 \ N/mm^2}$$

TORSION OF CIRCULAR SHAFTS

→ In workshops and factories, a turning force is always applied to transmit energy by rotation. This turning force is applied either to the rim of a pulley, keyed to the shaft or at any other suitable point at some distance from the axis of the shaft.

→ The product of this turning force and the distance between the point of application of the force and the axis of the shaft is known as Torque, Turning moment or Twisting moment. And the shaft is said to be subjected to Torsion.

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→ Due to this torque, every cross-section of the shaft is subjected to some shear stress.

Assumptions for shear stress in a Circular Shaft Subjected to Torsion.

Following assumptions are made, while finding out shear stress in a circular shaft subjected to torsion:

1. The material of the shaft is uniform throughout.
2. The twist along the shaft is uniform.
3. Normal cross sections of the shaft, which were plane and circular before the twist, remain plane and circular even after the twist.
4. All diameters of the normal cross section, which were straight before the twist, remain straight with their magnitude unchanged, after the twist.

The torque applied is small and the angle of twist is also small.

Strength of a Solid Shaft.

The term, strength of a Shaft means the maximum torque or power, it can transmit.

$$\boxed{T = \frac{\pi}{16} \times \tau \times D^3} \; N.mm$$

τ - Shear stress developed in the outermost layer of the Shaft in N/mm²

D - Diameter of the Shaft and is equal to 2R.

Problem:

1. A circular shaft of 50mm diameter is required to transmit torque from one shaft to another. Find the safe torque, which the shaft can transmit, if the shear stress is not to exceed 40MPa.

Given:

Diameter of Shaft (D) = 50mm.

Maximum Shear stress (τ) = 40MPa
= 40 N/mm²

Safe Torque,

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 40 \times 50^3$$

$$= 0.982 \text{ KN.m}$$

2. A Solid Steel Shaft is to transmit a torque of 10 KN.m. If the Shearing Stress is not to exceed 45 MPa. Find the minimum diameter of the Shaft.

Given, Torque, $T = 10 \text{ KN.m} = 10 \times 10^6 \text{ N.mm}$

$\tau = 45 \text{ MPa} = 45 \text{ N/mm}^2$.

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$10 \times 10^6 = \frac{\pi}{16} \times 45 \times D^3$$

$$D^3 = 1.132 \times 10^6$$

$$D = 1.04 \times 10^2$$

$$= 104 \text{ mm}$$

Strength of a Hollow Shaft

$$\boxed{T = \frac{\pi}{16} \times \tau \times \left(\frac{D^4 - d^4}{D}\right) N.mm}$$

D- external diameter of shaft
d - Internal diameter of shaft.

3. A Hollow Shaft of external and internal diameter of 80 mm and 50 mm is required to transmit torque from one end to the other. What is the safe torque it can transmit, if the allowable stress is 45 MPa?

Given: External diameter (D) = 80 mm
Internal diameter (d) = 50 mm
Allowable Shear stress (τ) = 45 MPa.

Torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau \left[\frac{D^4 - d^4}{D}\right]$$

$$= \frac{\pi}{16} \times 45 \times \left[\frac{80^4 - 50^4}{80}\right]$$

$$= 3.83 \times 10^6 \; N.mm$$

$$= 3.83 \; kN.m.$$

POWER TRANSMITTED BY A SHAFT

→ The main purpose of a shaft is to transmit power from one shaft to another in factories and workshops.

Workdone per minute = Force × Distance

$= T \times 2\pi N$

$= 2\pi NT$

Work done per second $= \frac{2\pi NT}{60}$ kN.m

Power transmitted = Workdone in kN.m per second

$$\boxed{P = \frac{2\pi NT}{60} \text{ kW}}$$

Note:

If the torque is in the N-m, then work done will also be in N-m and power will be in Watt (W)

4. A circular shaft of 60mm diameter is running at 150 r.p.m. If the shear stress is not to exceed 50MPa, find the power which can be transmitted by the shaft.

Given: Diameter of the shaft (D) = 60mm

Speed of the shaft (N) = 150 r.p.m

Maximum shear stress (τ) = 50MPa

$= 50 N/mm^2$

Torque transmitted by the shaft

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 50 \times 60^3 \ N.mm$$

$$= 2.12 \times 10^6 \ N.mm$$

$$T = 2.12 \ kN.m$$

Power which can be transmitted by the shaft,

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 150 \times 2.12}{60}$$

$$\boxed{P = 33.3 \ kW}$$

5. A solid circular shaft of 100mm diameter is transmitting 120 kW at 150 r.p.m. Find the Intensity of shear stress in the shaft?

Given;

Diameter of shaft (D) = 100 mm

Power transmitted (P) = 120 kW.

Speed of the shaft (N) = 150 r.p.m

We know that

$$T = \frac{\pi}{16} \times \tau \times D^3$$

To find T

$$P = \frac{2\pi N T}{60}$$

$$120 = \frac{2\pi \times 150 \times T}{60}$$

$$T = 7.64 \times 10^6 \text{ N.mm.}$$

Sub T in above formula.

$$7.64 \times 10^6 = \frac{\pi}{16} \times \tau \times D^3$$

$$\boxed{\tau = 39 \text{ N/mm}^2}$$

POLAR MOMENT OF INERTIA

The moment of inertia of a plane area, with respect to an axis perpendicular to the plane of the figure, is called polar moment of inertia with respect to the point, where the axis intersects the plane.

In a circular plane, this point is always the centre of the circle. We know that

$$\frac{\tau}{R} = \frac{C.\theta}{l} \rightarrow (1)$$

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$\Rightarrow \tau = \frac{16T}{\pi D^3}$$

sub τ in equ (1)

$$\frac{16T}{\pi D^3 \times R} = \frac{C.\theta}{l}$$

or

$$\frac{T}{\frac{\pi}{16} \times D^3 \times R} = \frac{C.\theta}{l}$$

$$\frac{T}{\frac{\pi}{32} \times D^4} = \frac{C \cdot \theta}{l}$$

$$\boxed{\frac{T}{J} = \frac{C \cdot \theta}{l}}$$

where

$\boxed{J = \frac{\pi}{32} \times D^4}$ → polar Moment of Inertia (circular)

The above equation can also be written as

$$\boxed{\frac{\tau}{R} = \frac{T}{J} = \frac{C \cdot \theta}{l}}$$

Polar moment of Inertia → Hollow circular

$$\boxed{J = \frac{\pi}{32} (D^4 - d^4)}$$

The term $\boxed{\frac{J}{R}}$ is known as torsional section modulus or polar modulus.

$Z_p = \frac{\pi}{16} D^3$ → solid shaft

$Z_p = \frac{\pi}{16D} (D^4 - d^4)$ → Hollow shaft

6. Calculate the maximum torque that a shaft of 12.5mm diameter can transmit, if the maximum angle of twist is 1° in a length of 1.5m. Take C = 70GPa

Given: D = 125mm, $\theta = 1° = \frac{\pi}{180}$ rad

Length of shaft = 1.5m = 1.5×10^3 mm.

$C = 70 GPa = 70 \times 10^3 \ N/mm^2$.

Polar moment of Inertia of solid circular shaft, $J = \frac{\pi}{32} \times D^4$

$= \frac{\pi}{32} \times 125^4$

$= 24.0 \times 10^6 \ mm^4$

Relation for torque transmitted by the shaft

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{T}{24 \times 10^6} = \frac{(70 \times 10^3)\ \pi/180}{1.5 \times 10^3}$$

$$\boxed{T = 19.5 \ kN.m}$$

9. Find the maximum torque, that can be safely applied to a shaft of 80mm diameter. The permissible angle of twist is 1.5° in a length of 5m and shear stress not to exceed 42MPa. Take C = 84GPa.

Given.

$D = 80mm$, $\theta = 1.5^\circ = 1.5 \times \frac{\pi}{180}$ rad.

$l = 5m = 5 \times 10^3 mm$, $\tau = 42MPa$

$C = 84GPa = 84 \times 10^3 N/mm^2$.

To find

Torque based on shear stress & angle of twist.

1) Torque based on shear stress

$$T = \frac{\pi}{16} \times \tau \times D^3$$

$$= \frac{\pi}{16} \times 42 \times 80^3$$

$$= 4.22 \times 10^6 N.mm.$$

2) Torque based on angle of twist

$$\frac{T}{J} = \frac{C.\theta}{l}.$$

$$\frac{T}{J} = \frac{(84\times10^3)\times(15\pi/180)}{5\times10^3} \quad \Big| \; J = \frac{\pi}{32}(D)^4$$

$$\frac{T}{\frac{\pi}{32}(80)^4} = 0.44$$

$$T = 1.77\times10^6 \text{ N.mm.}$$

We shall apply a torque of 1.77×10^6 N-mm

(i.e lesser of the two values)

COMBINED BENDING AND TORSION

→ when a shaft is transmitting torque or power, it is subjected to shear stress.

→ At the same time the shaft is also subjected to bending moment due to gravity.

→ Due to bending moment, bending stresses are also set up in the shaft.

→ Hence each particle in a shaft is subjected to shear stress and

Date
Page

bending stress :-

→ For design purpose it is necessary to find principle stresses, Maximum shear stress and strain energy.

→ The principal stresses and maximum shear stress when a shaft is subjected to bending and torsion, are obtained as

Shear Stress

$$\tau = \frac{T}{J} \times R$$

Bending Stress

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma_1 = \frac{M \times y}{I}$$

Bending stress and shear stress is maximum at a point on the surface of the shaft. $R = D/2$ & $y = D/2$

$\sigma_b = \frac{32M}{\pi D^3}$,	$\tau = \frac{16 T}{\pi D^3}$
$\tan 2\theta = \frac{T}{M}$	

Major principal stress [circular shaft]

$$= \frac{16}{\pi D^3}\left(M + \sqrt{M^2+T^2}\right)$$

Minor principal stress [circular shaft]

$$= \frac{16}{\pi D^3}\left(M - \sqrt{M^2+T^2}\right)$$

Maximum shear stress [circular shaft]

$$= \frac{16}{\pi D^3}\sqrt{M^2+T^2}$$

<u>Hollow Shaft</u>

Major principal stress

$$= \frac{16 D_o}{\pi (D^4 - d^4)}\left(M + \sqrt{M^2+T^2}\right)$$

Minor principal stress

$$= \frac{16 D}{\pi (D^4 - d^4)}\left(M - \sqrt{M^2-T^2}\right)$$

Maximum shear stress

$$= \frac{16 D}{\pi (D^4 - d^4)}\left(\sqrt{M^2+T^2}\right)$$

8. A solid shaft of diameter 80mm is subjected to a twisting moment of 8 MN-mm and a bending moment of 5MN-mm at a point. Determine (i) principal stresses and (ii) position of the plane on which they act.

Given.

Diameter of shaft $D = 80 mm$

Twisting moment $T = 8 MN\text{-}mm$

$= 8 \times 10^6 N\text{-}mm$

Bending moment $M = 5 MN.mm$

$= 5 \times 10^6 N.mm$

Major principal stress $= \frac{16}{\pi D^3}(M + \sqrt{M^2 + T^2})$

$= 143.57 N/mm^2$

Minor principal stress $= \frac{16}{\pi D^3}(M - \sqrt{M^2 + T^2})$

$= 44.1 N/mm^2$

Position of plane

$\tan 2\theta = \frac{T}{M}$

$2\theta = \tan^{-1}(1.6)$

$\theta = 28^\circ 59.84'$

9. The maximum allowable Shear Stress in a hollow shaft of external diameter equal to twice the internal diameter, is 80 N/mm². Determine the diameter of the shaft if it is subjected to a torque of 4×10^6 N-mm and a bending moment of 3×10^6 N-mm.

Given:

Maximum shear stress = 80 N/mm²

Torque, $T = 4\times10^6$ N.mm

Bending Moment, $M = 3\times10^6$ N.mm

D - External dia

d - Internal dia.

Then $D = 2d$

Maximum Shear stress

$$= \frac{16D}{\pi(D^4-d^4)}\left(\sqrt{M^2+T^2}\right)$$

$$80 = \frac{16D}{\pi\left[D^4-\left(\frac{D}{2}\right)^4\right]}\left(\sqrt{(3\times10^6)^2+(4\times10^6)^2}\right)$$

$D = 69.78$ mm

$d = D/2 = 34.89$ mm

09-01-2020

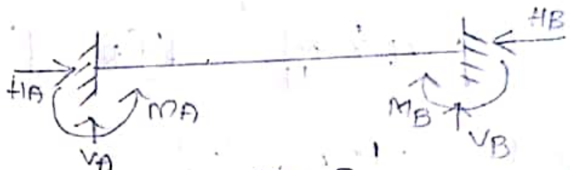
FIXED & CONTINUOUS BEAMS

Statically Indeterminate structure:-

If the given structure is not analysed by only using of equilibrium conditions and requires any boundary conditions to Fully analysed the given structure is statically Indeterminate structure. [$r<R$]

The no. of required boundary conditions is called degree of Indeterminacy.

Ex:- Fixed beam, proped cantilever beam, continuous beam



⇒ r = no. of equilibrium equations.

R = no. of unknown support reactions

$r=3$,

$r=3$

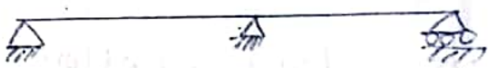
$R=6$

$R-r=6-3=3$ = Degree of Indeterminancy

⇒ For statically indeterminancy structure ⇒ $r<R$

⇒ For statically determinancy structure ⇒ $r\geq R$

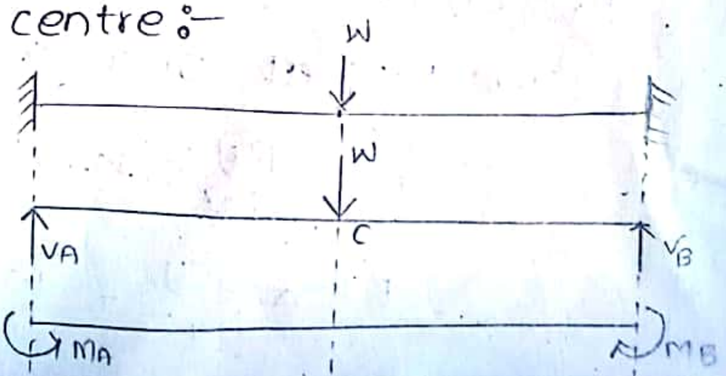
⇒ $r=3$, $R=5$



$R-r=5-3$

$=2$ Degree of Indeterminancy.

Case(i): Fixed beam carries a point load at the centre:-



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Integrate the above equation, w.r.t. "x", we get

$$EI\cdot\frac{dy}{dx} = \frac{w}{2}\cdot\frac{x^2}{2} - \frac{wL}{8}\cdot x + C_1 \longrightarrow (2)$$

From boundary conditions

$x=0,\ \frac{dy}{dx}=0$ apply in eq (2).

$\boxed{\therefore C_1 = 0}$ put eq (2), we get

$$\boxed{EI\frac{dy}{dx} = \frac{w}{2}\frac{x^2}{2} - \frac{wL}{8}x} \longrightarrow (3)$$

again Integrate on both sides w.r.t. "x"

we get

$$EI\cdot y = \frac{w}{4}\frac{x^3}{3} - \frac{wL}{8}\cdot\frac{x^2}{2} + C_2 \longrightarrow (4)$$

From boundary conditions

$x=0,\ y=0$ put eq (4), we get

$\boxed{\therefore C_2 = 0.}$

$C_2=0$ put eq (4), we get

$$\boxed{EI\cdot y = \frac{wx^3}{12} - \frac{wLx^2}{16}} \longrightarrow (5)$$

2 | 96, 64
2 | 48, 32
2 | 24, 16
2 | 12, 8
2 | 6, 4
3, 2

Max. deflection at centre $x = l/2$

$$EI\,y_{max} = \frac{w(l)^3}{12(2)^3} - \frac{w(l)(l)^2}{16\times(2)^2} \Rightarrow$$

$32\times6 = 192$

$$\Rightarrow \frac{wl^3}{96} - \frac{wl^3}{64} \Rightarrow \frac{2wl^3 - 3wl^3}{192}$$

$$\Rightarrow -\frac{wL^3}{192}$$

$$\boxed{\therefore y_{max} = \frac{wL^3}{192\,EI}}$$

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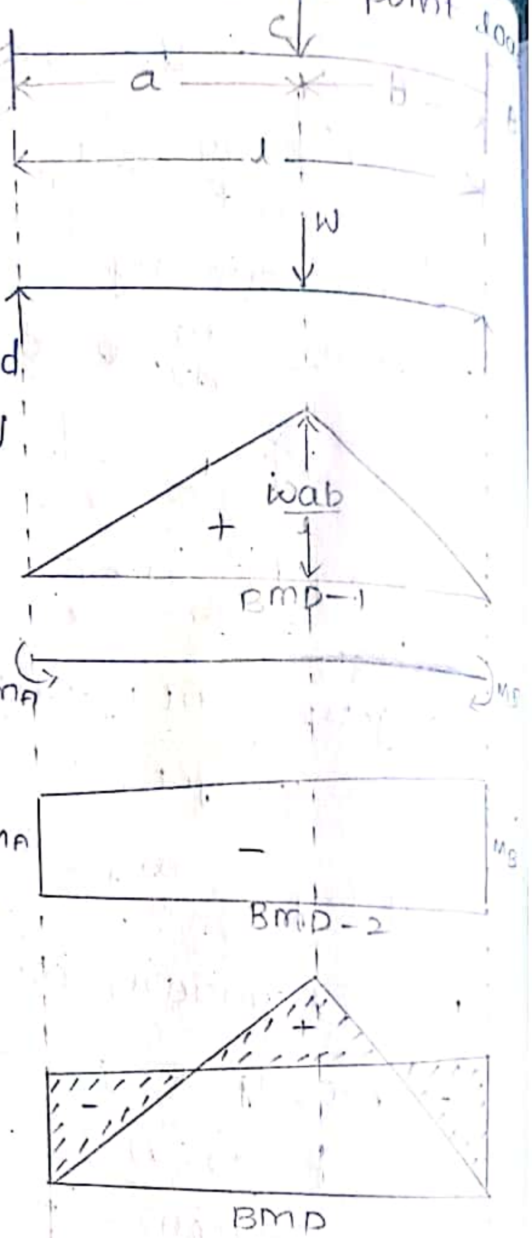
case(ii): Fixed beam carries a eccentric point load



1. consider a simply supported beam carries only vertical forces.

2. The beam is subjected to end moments only

3. a = area of BMD due to vertical forces

a' = area of BMD due to end moments

$R_A + R_B = W$

$R_A(0) - W(a) + R_B(l) = 0$

$\boxed{R_B = \frac{Wa}{l}}$

$\boxed{R_A = \frac{Wb}{l}}$

$a = a'$

$BM_{max} \Rightarrow \frac{Wb}{l} \times a$

$a = \frac{1}{2} \times l \times \frac{Wab}{l}$

$\therefore a = \frac{Wab}{2}$

$a' = \frac{(a+b)}{2} \times h$

$\Rightarrow \left[\frac{M_A + M_B}{2}\right] \times l$

$a = a'$

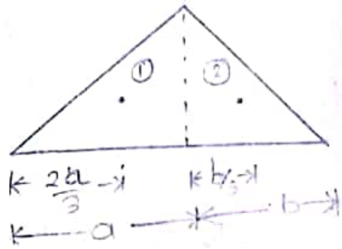
$\frac{Wab}{2} = \left[\frac{M_A + M_B}{2}\right] l$

$M_A + M_B = \frac{Wab}{l} \longrightarrow (1)$

From

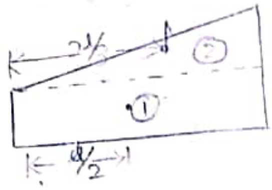
$$a = a'$$
$$\bar{x} = \bar{x}'$$

x = Distance of C-G of BM diagram due to vertical forces



$$a x = a' \bar{x}'$$

$$\left[\frac{1}{2} \times a \times \frac{wab}{l} \times \frac{2a}{3}\right] + \left[\frac{1}{2} \times b \times \frac{wab}{l} \times \left(a + \frac{b}{3}\right)\right] =$$



$$\left[(M_A \times l) \times \frac{l}{2}\right] + \left[\frac{1}{2} \times l \times (M_B - M_A) \times \frac{2l}{3}\right]$$

$$\left[\frac{wa^3b}{3l}\right] + \left[\frac{wa^2b^2}{2l} + \frac{b^3 a w}{6l}\right] = M_A \times \frac{l^2}{2} + (M_B - M_A)\frac{l^2}{3}$$

$$\frac{wa^3b}{3l} + \frac{wa^2b^2}{2l} + \frac{wab^3}{6l} = M_A\frac{l^2}{2} + M_B\frac{l^2}{3} - M_A\frac{l^2}{3}$$

$$\frac{2wa^3b + 3wa^2b^2 + wab^3}{\cancel{6}l} = \frac{3M_A l^2 + 2M_B l^2 - 2M_A l^2}{\cancel{6}}$$

$$\frac{w(2a^3b + 3a^2b^2 + ab^3)}{l} = l^2[M_A + 2M_B]$$

$$\frac{w\,ab[2a^2 + 3ab + b^2]}{l} = l^2[M_A + 2M_B]$$

$$\frac{w\,ab[a+b][2a+b]}{l} = (M_A + 2M_B)\,l^2$$

$(a+b) = l$

$$\frac{wab\,\cancel{l}(2a+b)}{\cancel{l}} = [M_A + 2M_B]\,l^2$$

$$\frac{wab(2a+b)}{l^2} = M_A + 2M_B \longrightarrow (2)$$

then $M_A = \frac{wab}{l} - \frac{wa^2b}{l^2} \Rightarrow \frac{wab[l-a]}{l^2}$

$$M_A = \frac{wab^2}{l^2}$$

$$\cancel{M_A} + 2M_B = \frac{wab(2a+b)}{l^2}$$

$$\cancel{M_A} + M_B = \frac{wab}{l}$$

$$M_B = \frac{wab}{l^2}\left((2a+b) - l\right)$$

$$M_B = \frac{wab[a]}{l^2}$$

$$\boxed{\therefore M_A = \frac{wab^2}{l^2}}$$

$$\boxed{\therefore M_B = \frac{wa^2b}{l^2}}$$

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position of point of contraflexure :-

$$M_x = R_A \times x - M_A - (M_B - M_A)\frac{x}{l}$$

$$M_x = \frac{wb}{l}x - \frac{wab^2}{l^2} - \left[\frac{wa^2b}{l^2} - \frac{wab^2}{l^2}\right]\frac{x}{l}$$

$$0 = \frac{wb}{l}\left[x - \frac{ab}{l} - \frac{a^2x}{l^2} + \frac{ab \cdot x}{l^2}\right]$$

$$x - \frac{a^2x}{l^2} + \frac{ab}{l} + \frac{abx}{l^2} = 0$$

$$x\left[1 - \frac{a^2}{l^2} + \frac{ab}{l^2}\right] = \frac{ab}{l}$$

$$x\left[\frac{l^2 - a^2 + ab}{l^{\cancel{2}}}\right] = \frac{ab}{\cancel{l}}$$

$$x = \frac{abl}{l^2 - a^2 + ab}$$

$$x = \frac{abl}{(a+b)^2 - a^2 + ab} = \frac{abl}{\cancel{a^2} + 2ab + b^2 - \cancel{a^2}}$$

$$x = \frac{abl}{3ab + b^2} = \frac{\cancel{b}(al)}{\cancel{b}(3a+b)}$$

$$\boxed{\therefore x = \frac{al}{3a+b}}$$ from "A"

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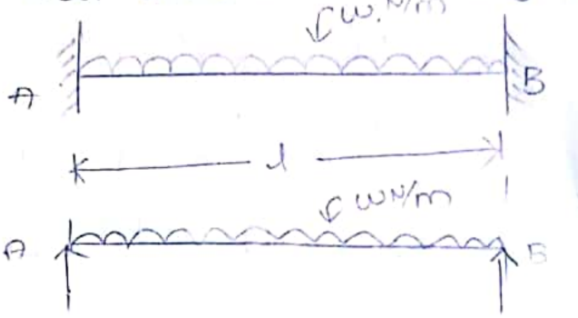
24/01/2020
04/02/2020

Fixed beam carries udl load on entire length:



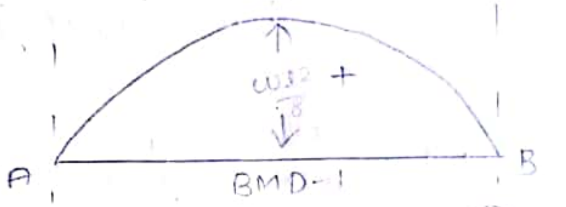
$R_A + R_B = wl$

$R_A(0) - w(l)\left(\frac{l}{2}\right) + lR_B = 0$

$R_B = \frac{wl}{2}$

$R_A = \frac{wl}{2}$

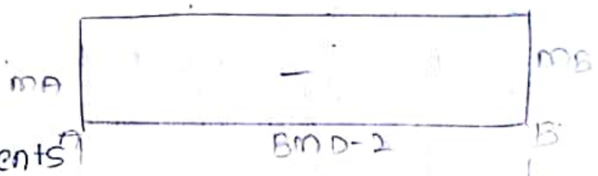
(1) BM due to udl load



$BM_A = 0$

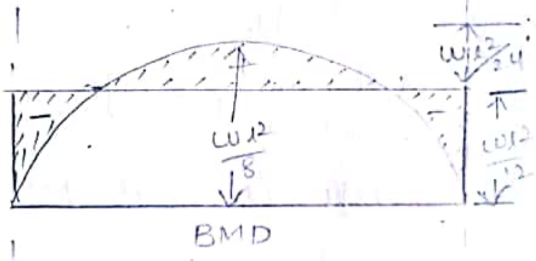
$BM_B = 0$

$BM_{max} = \frac{wl}{2} \times \frac{l}{4} \Rightarrow \frac{wl^2}{8}$



(2) consider beam subjected to end moments only

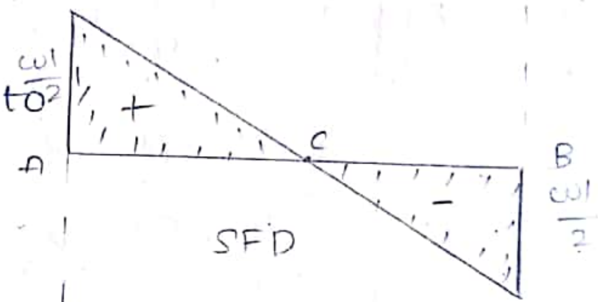
$M_A = M_B$



(3) $A_1 = A_2$

A_1 = Area of BMD due to vertical loads

A_2 = Area of BMD due to end moments



$A_1 = \frac{2}{3} \times L \times \frac{wL^2}{8}$

$A_2 = L \times M_B$

$\frac{2}{3} \cancel{L} \frac{wL^2}{8_4} = \cancel{L} M_B$

$M_B = \frac{wL^2}{12} = M_A$

$\boxed{\therefore M_A = M_B = \frac{wL^2}{12}}$

Position of point of contraflexure:—

$M_x = R_A x - \frac{wx^2}{2} - M_A$

$\frac{wl}{2} x - \frac{wl^2}{12} - \frac{wx^2}{2} = 0$

$\frac{6wlx - wl^2 - 6wl^2}{12} = 0$

$6x^2 - 6xl + l^2 = 0$

$\boxed{x = 0.212\,l}$ and

$\boxed{x = 0.788\,l}$

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$$M_x = R_A \times x - M_A - w x \times \frac{x}{2}$$

$$M_x = \frac{wl}{2}x - \frac{wl^2}{12} - \frac{wx^2}{2} \longrightarrow (1)$$

$$EI\frac{d^2y}{dx^2} = -M \longrightarrow (2)$$

from (1) & (2)

$$EI.\frac{d^2y}{dx^2} = -\left[\frac{wl}{2}x - \frac{wl^2}{12} - \frac{wx^2}{2}\right]$$

Integrate above eq. w.r.t. to x

$$EI\frac{dy}{dx} = -\left[\frac{wl}{2}\frac{x^2}{2} - \frac{wl^2}{12}.x - \frac{wx^3}{2\times3} + C_1\right] \longrightarrow (3)$$

$\frac{dy}{dx} = 0 ; x = 0$ sub. in eq (3), we get

$$\boxed{C_1 = 0}$$

Again Integrate above eq w.r.t. 'x', we g

$$EI.y = -\left[\frac{wl}{4}\frac{x^3}{3} - \frac{wl^2}{12}\frac{x^2}{2} - \frac{wx^4}{6\times4} + C_1x + C_2\right]$$

$y = 0 ; x = 0$ sub. eq (4), we get

$$\boxed{\therefore C_2 = 0}$$

$$EI.\frac{dy}{dx} = -\left[\frac{wlx^2}{4} - \frac{wl^2}{12}x - \frac{wx^3}{6}\right] \longrightarrow (5)$$

$$y = -\left[\frac{wlx^3}{12} - \frac{wl^2x^2}{24} - \frac{wx^4}{24}\right] \longrightarrow (6)$$

5 & (6) eq will gives the slope & deflection at any section in the given beam.

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The max. deflection at centre

then sub. $x = l/2$ in eq (6)

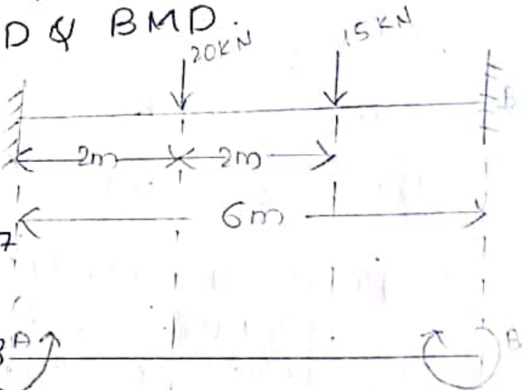
$$EI\, y_{max} = -\left[\frac{wl(l/2)^3}{12} - \frac{wl^2(l/2)^2}{24} - \frac{w(l/2)^4}{24}\right]$$

$$EI\, y_{max} = -\left[\frac{wl \cdot l^3}{96} - \frac{wl^2 l^2}{24 \times 4} - \frac{wl^4}{16 \times 24}\right]$$

$$EI\, y_{max} = -\left[\cancel{\frac{wl^4}{96}} - \cancel{\frac{wl^4}{96}} - \frac{wl^4}{384}\right] \Rightarrow -\left[-\frac{wl^4}{384}\right]$$

$$\boxed{\therefore y_{max} = \frac{wl^4}{384\, EI}}$$

1. A fixed beam of span 6m. it carries a point loads of 20 KN and 15 KN at a distance of 2m & 4m from the end "A". Find the support moments and draw the SFD & BMD.



consider only 20 KN force

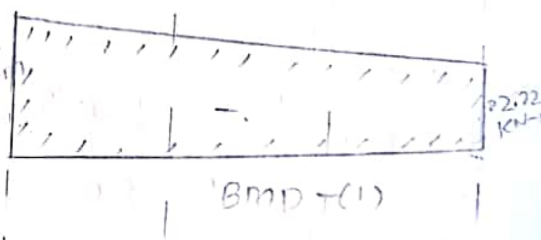
$$M_A = \frac{wab^2}{l^2} \Rightarrow \frac{20(2)(4)^2}{6^2} \Rightarrow 17.77$$

$$m_B = \frac{wa^2b}{l^2} \Rightarrow \frac{20(2)^2 \times 4}{6^2} \Rightarrow 8.88$$

consider only 15 KN force

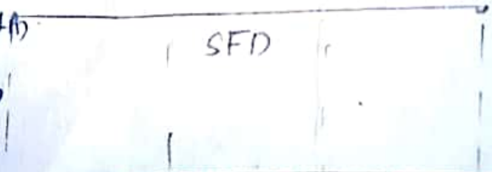
$$m_A = \frac{wab^2}{l^2} \Rightarrow \frac{15(4)(2)^2}{6^2} \Rightarrow 6.67$$

$$m_B = \frac{wa^2b}{l^2} \Rightarrow \frac{15(4)^2(2)}{6^2} \Rightarrow 13.33$$



$$\Sigma m_A = 17.77 + 6.67 \Rightarrow 24.44\,KN\text{-}m$$

$$\Sigma m_B = 8.88 + 13.34 \Rightarrow 22.22\,KN\text{-}m$$



$R_A + R_B = 20 + 15$

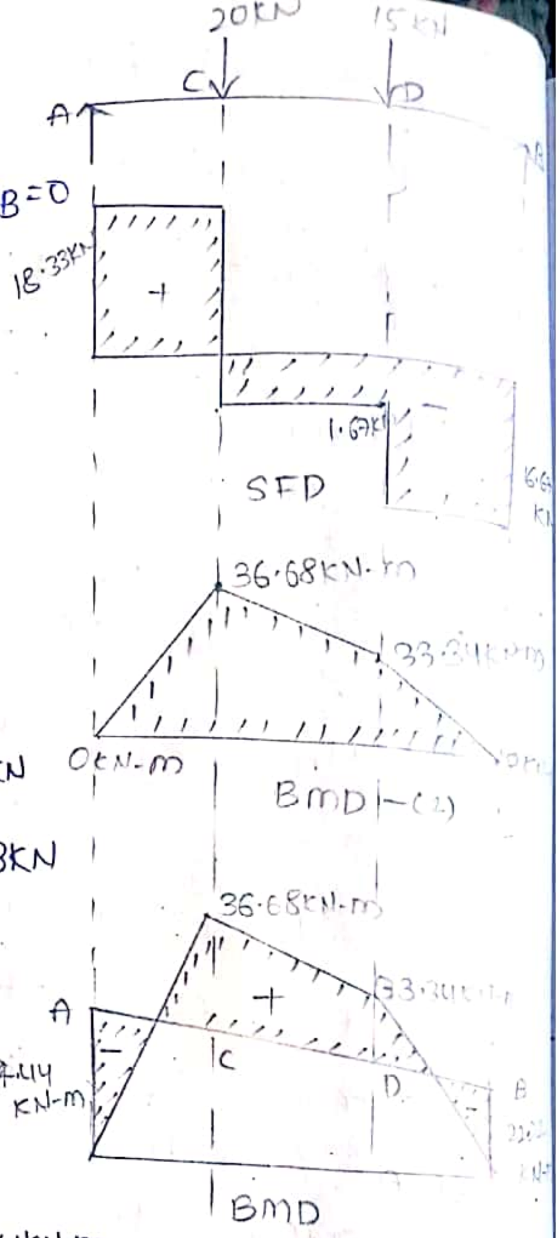
$\Sigma M_A = 0$

$R_A(0) - 20(2) - 15(4) + 6R_B = 0$

$6R_B = 100$

$\boxed{\therefore R_B = 16.67\,KN}$

$\boxed{R_A = 18.33\,KN}$



SFD:-

$SF_B = -R_B \Rightarrow -16.67\,KN$

$SF_D = -16.67 + 15 \Rightarrow -1.67\,KN$

$SF_C = -1.67 + 20 \Rightarrow 18.33\,KN$

$SF_A = 18.33\,KN.$

BMD:-

$BM_A = 0$

$BM_B = 0$

$BM_D = 16.67 \times 2 \Rightarrow 33.34\,KN\text{-}m$

$BM_C \Rightarrow 16.67 \times 4 - 15 \times 2 \Rightarrow 36.68\,KN\text{-}m$

Point of contraflexure:-

1st P.O.C. is b/w A & C

Consider a section at a distance 'x' from "A".

$$M_x = R_A \cdot x - M_A = 0$$

$$18.33x - 24.44 = 0$$

$$x = \frac{24.44}{18.33}$$

$\boxed{\therefore x = 1.33\,m}$ from A.

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2nd P.O.C. is in b/w D&B
consider a section at a distance x from "B"

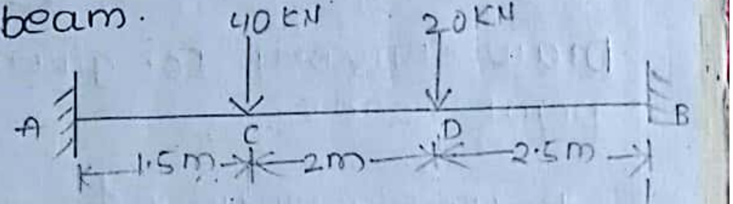
$$M_x = R_B \times x - M_B = 0$$

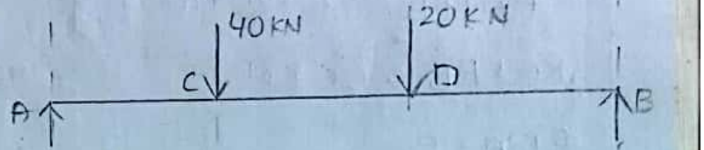
$$16.67x - 22.22 = 0$$

$$x = \frac{22.22}{16.67}$$

$\boxed{\therefore x = 1.33m}$ from "B"

Draw SFD & BMD
for given fixed beam.





3) $R_A + R_B = 40 + 20$

$R_A + R_B = 60$ KN $\longrightarrow (1)$

$\Sigma M_A = 0$

$R_A(0) - 40(1.5) - 20(3.5) + 6R_B = 0$

$\boxed{\therefore R_B = 21.67 \text{ KN}}$

$\boxed{\therefore R_A = 38.33 \text{KN}}$

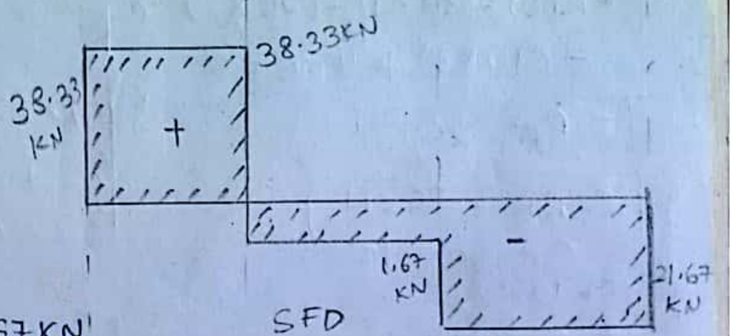
SFD:-

$SF_B = -21.67$ KN

$SF_D = -21.67 + 20 \Rightarrow -1.67$ KN

$SF_C \Rightarrow -21.67 + 20 + 40 \Rightarrow +38.33$ KN

$SF_A = 38.33$ KN



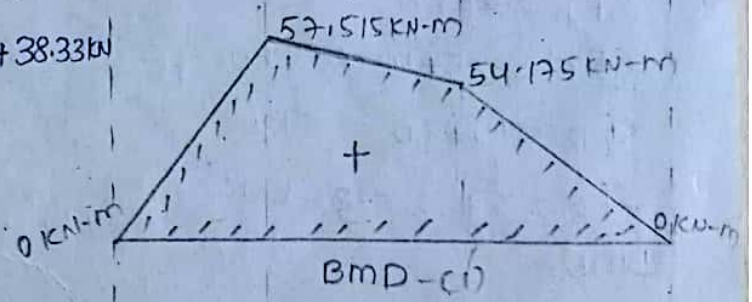
BMD:-

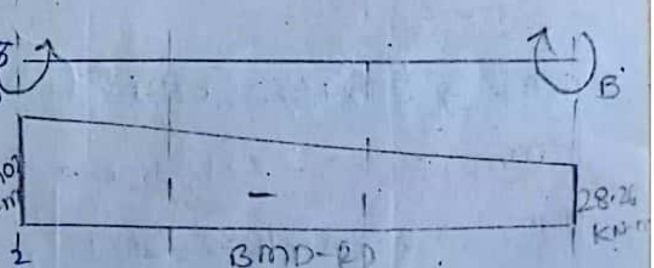
$BM_A = 0$

$BM_B = 0$

$BM_D \Rightarrow 21.67 \times 2.5 \Rightarrow 54.178$ KN-m

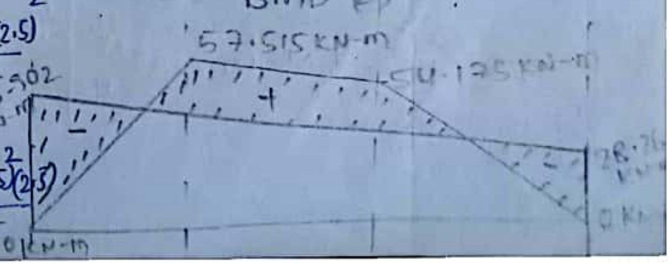
$BM_C \Rightarrow 21.67 \times 4.5 - 20 \times 2$
$\Rightarrow 57.515$ KN-m





$\Sigma M_A = \Sigma \frac{Wab^2}{l^2} \Rightarrow \frac{40(1.5)(4.5)^2}{6^2} + \frac{20(3.5)(2.5)^2}{6^2}$
$\Rightarrow 45.902$ KN-m

$\Sigma M_B = \Sigma \frac{Wa^2b}{l^2} \Rightarrow \frac{40(1.5)^2(4.5)}{6^2} + \frac{20(3.5)^2(2.5)}{6^2}$
$\Rightarrow 28.26$ KN-m



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point of contraflexure :-

(1) Consider a section at a distance "x" from A

$m_x = R_A x - m_A$

$38.33x - 45.902 = 0$

$x = 1.19\,m$ from support "A"

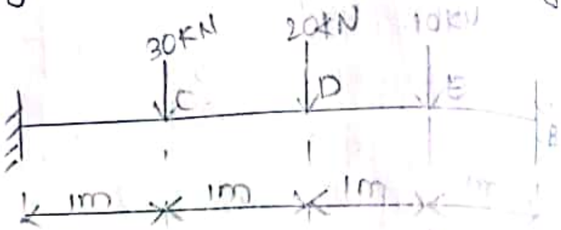
(2) Consider a section at a distance 'x' from B

$m_x \Rightarrow R_B x - m_B$

$21.67x - 28.26 = 0$

$\boxed{x = 1.30m}$ from support "B"

Draw SFD & BMD for given fixed beam carries point loads.



$R_A + R_B = 30+20+10$

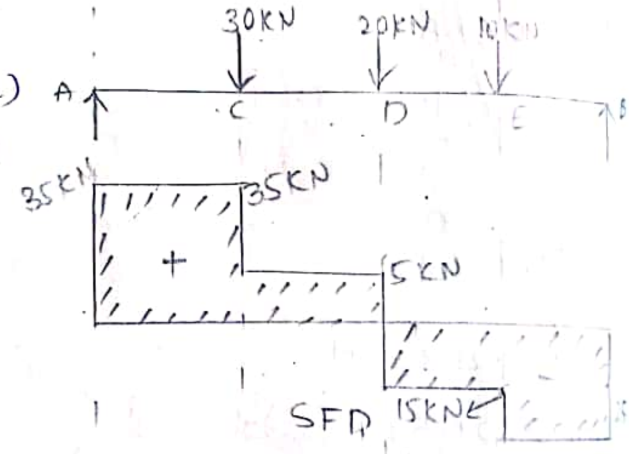
$R_A + R_B = 60\,KN$

$\Sigma m_A = 0$

$R_A.(0) - (30\times1) - (20\times2) - (10\times3) + 4R_B = 0$

$\boxed{\therefore R_B = 25\,KN}$

$\boxed{\therefore R_A = 35\,KN}$



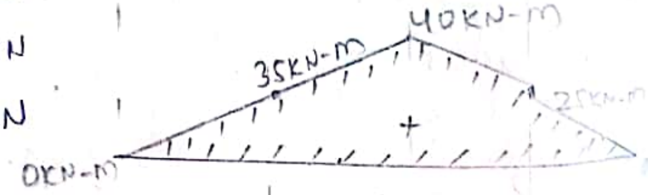
SFD:-

$SF_B = -25\,KN$

$SF_E = -25+10 \Rightarrow -15\,KN$

$SF_D = -15+20 \Rightarrow 5\,KN$

$SF_C = 5+30 \Rightarrow 35\,KN$



BMD:-

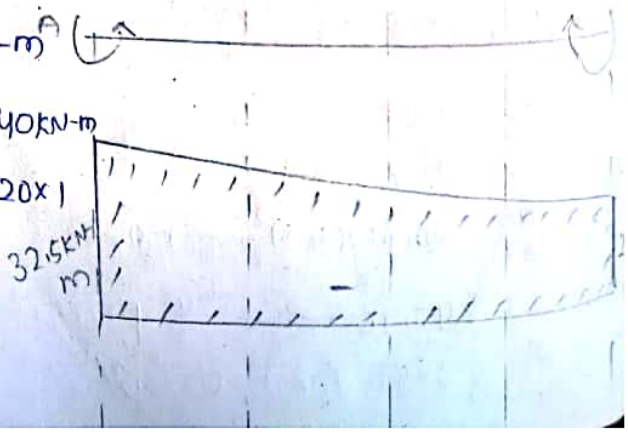
$BM_B = 0$

$BM_E \Rightarrow 25\times1 \Rightarrow 25\,KN\text{-}m$

$BM_D \Rightarrow 25\times2 - 10\times1 \Rightarrow 40\,KN\text{-}m$

$BM_C \Rightarrow 25\times3 - 10\times2 - 20\times1$

$\Rightarrow 35\,KN\text{-}m$

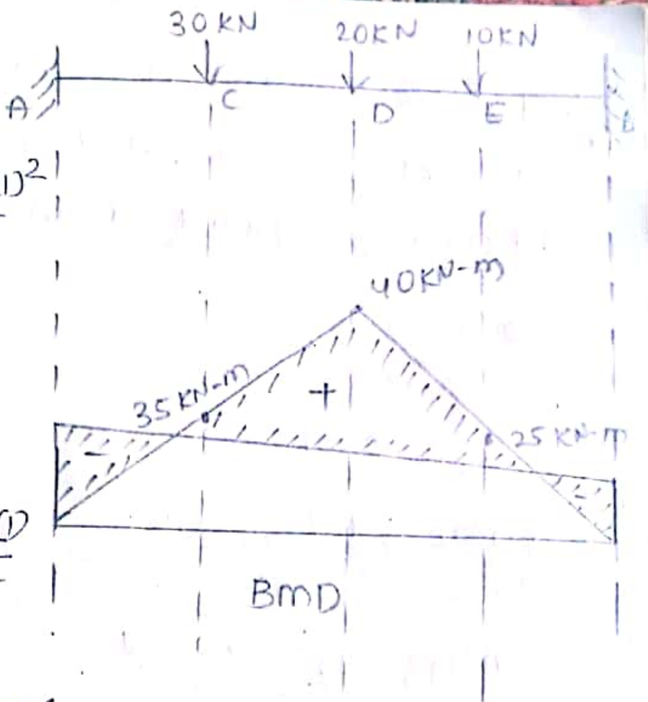


$BM_A = 0$

$\Sigma M_A = \Sigma \frac{Wab^2}{l^2} \Rightarrow$

$\frac{30(1)(3)^2}{4^2} + \frac{20(2)(2)^2}{4^2} + \frac{10(3)(1)^2}{4^2}$

$\Rightarrow 28.75$ KN-m



$\Sigma M_B = \Sigma \frac{Wa^2b}{l^2} \Rightarrow$

$\frac{30(1)^2(3)}{4^2} + \frac{20(2)^2(2)}{4^2} + \frac{10(3)^2(1)}{4^2}$

$\Rightarrow 21.25$ KN-m

Point of contraflexure:-

(1) consider a section at a distance 'x' from A

$M_x = R_A x - M_A$

$35x - 28.75 = 0$

$\boxed{\therefore x = 0.82m}$ from "A"

(2) consider a section at a distance 'x' from B

$M_x = R_B x - M_B$

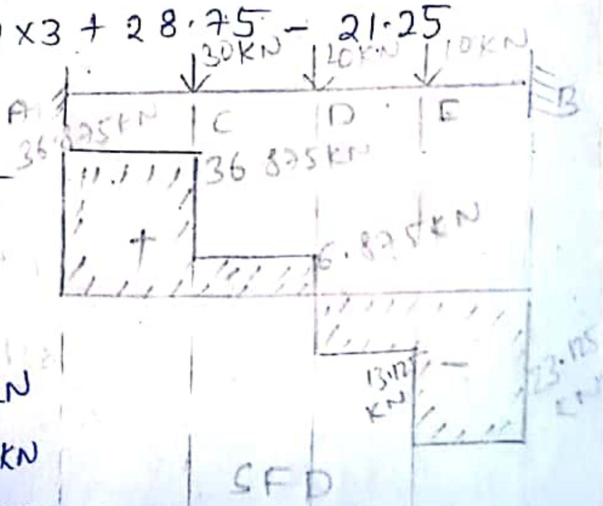
$25x - 21.25 = 0$

$\boxed{\therefore x = 0.85m}$ from "B"

$R_A + R_B = 60$ KN

$R_A(0) - 30\times1 - 20\times2 - 10\times3 + 28.75 - 21.25 + 4R_B = 0$

$R_B = \frac{92.5}{4} \Rightarrow 23.125$



SFD:- $R_A = 36.875$

$S_{RB} = -23.125$ KN

$SF_E \Rightarrow -23.125+10 \Rightarrow -13.125$ KN

$SF_D \Rightarrow -23.125+10+20 \Rightarrow 6.875$ KN

$SF_C \Rightarrow -23.125+10+20+30 \Rightarrow 36.875$ KN

$SF_A \Rightarrow 36.875$ KN

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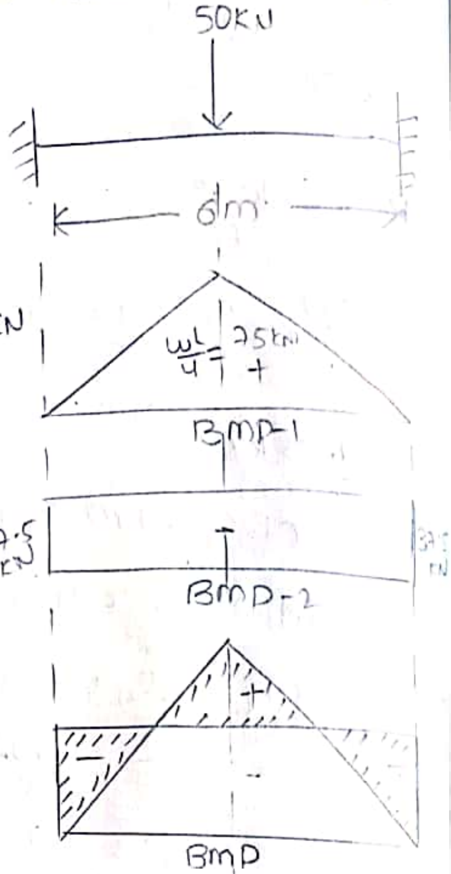
03/02/2020

1. To determine the Fixed End Moment and deflection of fixed Beam span of 6m and having point load of 50KN at centre.



Given

$W = 50KN$

$l = 6m$

$M_A = \frac{WL}{8} \Rightarrow \frac{50 \times 6}{8} \Rightarrow 37.5KN$

$M_A = M_B = 37.5KN$

$y_{max} = \frac{WL^3}{192EI} \Rightarrow \frac{50(6)^3}{192EI}$

$\Rightarrow \frac{56.25}{EI} m$

04/02/2020

Calculate deflection and fixed end moment for fixed beam carrying udl of 9 KN/m with span 5m take $E = 1 \times 10^7$ KN/m² and $I = 4 \times 10^{-4}$ m⁴

$M_A = M_B = \frac{wl^2}{12} = \frac{\cancel{9}^{3}(5)^2}{\cancel{12}_{4}} \Rightarrow \frac{75}{4}$

$\boxed{\therefore M_A = 18.75 KN\text{-}m}$

$y_c = \frac{wl^4}{384EI} \Rightarrow \frac{9 \times 5^4}{384 \times 1 \times 10^7 \times 4 \times 10^{-4}}$

$\Rightarrow 3.66 \times 10^{-3} m$

$\Rightarrow 3.66 mm$

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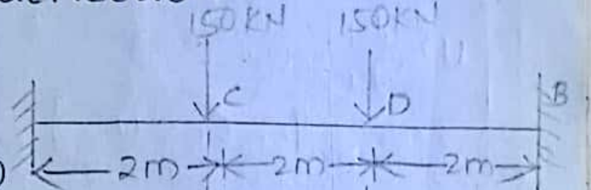
5-02-2020

For given fixed beam a fixed end moments SFD & BMD c, max deflection.

Macaulay's Method

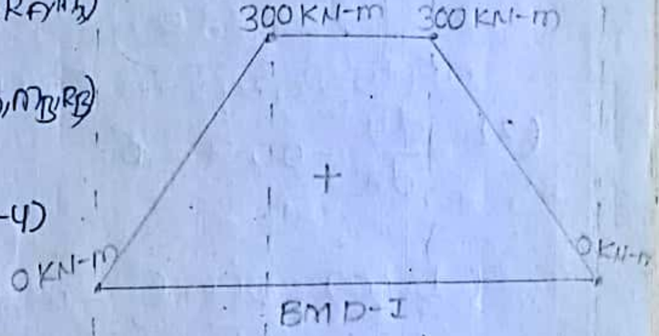
Boundary condition



$0 \le x < 2$ = 2 terms (M_A, R_A)

$2 \le x \le 4$ = 3 terms (M_A, R_A, M_3)

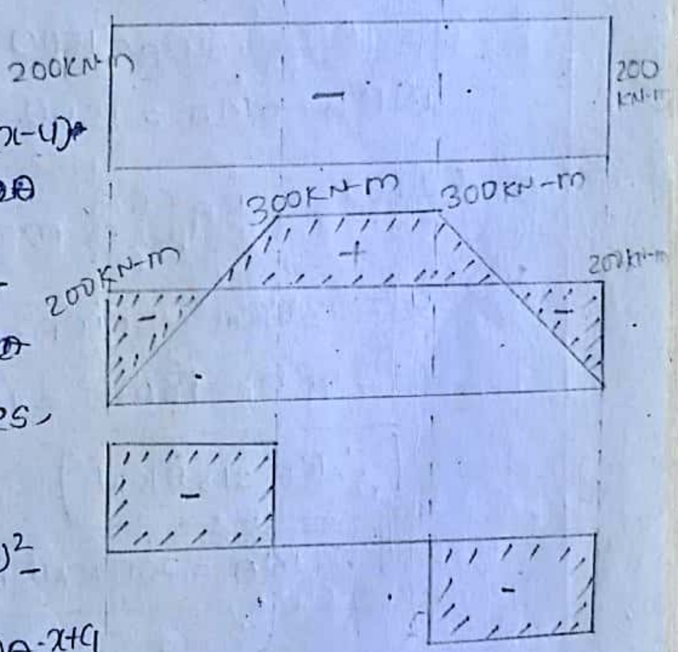
$4 \le x \le 6$ = 4 terms (M_A, R_A, M_B, R_B)



$-M_x + R_A x - 150(x-2) - 150(x-4)$

$-M_A = 0$

$EI\dfrac{d^2y}{dx^2} = M_x$



$M_x = R_A \times x - M_A + 150(x-2) - 150(x-4)$

$EI\dfrac{d^2y}{dx^2} = R_A \times x - M_A - 150(x-2) - 150(x-4)$

Integrate on both sides, we get

$EI\dfrac{dy}{dx} = R_A \cdot \dfrac{x^2}{2} - 150\dfrac{(x-2)^2}{2} - 150\dfrac{(x-4)^2}{2} - M_A \cdot x + C_1 \rightarrow (1)$

$EI \cdot y = R_A \cdot \dfrac{x^3}{2\times3} - \dfrac{150}{2} \cdot \dfrac{(x-2)^3}{3} - 150\dfrac{(x-4)^3}{2\times3} - M_A \dfrac{x^2}{2} + C_1 x + C_2 \rightarrow (2)$

from boundary conditions.

$\dfrac{dy}{dx} = 0$, $x = 0$ then apply in eq(1)

$EI\dfrac{dy}{dx} = R_A \dfrac{x^2}{2} - M_A \cdot x + C_1 \,\Big|\, -150\dfrac{(x-2)^2}{2} \,\Big|\, -150\dfrac{(x-4)^2}{2} \rightarrow (1)$

Integrate on both sides, we get

$EI \cdot y = R_A \cdot \dfrac{x^3}{6} - M_A x^2 + C_1 x + C_2 \,\Big|\, -150\dfrac{(x-2)^3}{6} \,\Big|\, -\dfrac{150(x-4)^3}{2\times3} \rightarrow (2)$

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Apply Boundary conditions in eq(1)&(2)

(1) $\frac{dy}{dx} = 0, \quad x = 0$

$$\boxed{\therefore C_1 = 0}$$

(2) $y = 0, \quad x = 0$

$$\boxed{\therefore C_2 = 0}$$

Apply C_1 & C_2 in eq(1)&(2)

(3) $\frac{dy}{dx} = 0, \quad x = 6$

$$EI\cdot(0) = R_A\frac{6^2}{2} - M_A(6) + 0 - \frac{150(6-2)^2}{2} - \frac{150(6-4)^2}{2}$$

$$0 = 18R_A - 6M_A - 1500$$

$$18R_A - 6M_A = 1500 \longrightarrow (3)$$

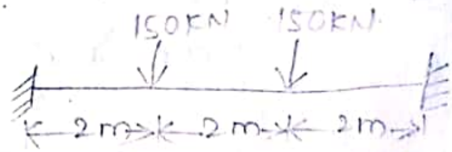
$$EI(0) = R_A\frac{(6)^3}{6} - M_A\frac{(6)^2}{2} + 0 + 0 - \frac{150(6-2)^3}{6} - \frac{150(6-4)^3}{6}$$

$$0 = 36R_A - 36M_A - 1800$$

$$36R_A - \cancel{78}M_A = 1800 \longrightarrow (4)$$

$$\boxed{\therefore R_A = 150\,KN}$$

$$\boxed{\therefore M_A = 200\,KN\text{-}m}$$



$$R_A + R_B = 150 + 150$$

$$R_A + R_B = 300$$

$$R_B = 300 - 150$$

$$\boxed{\therefore R_B = 150\,KN}$$

$\Sigma M_A = 0$

$$M_A + R_A(0) - 150(2) - 150(4) + 6R_B - M_B = 0$$

$$200 - 300 - 600 + 900 = M_B$$

$$\boxed{\therefore M_B = 200\,KN\text{-}m}$$

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SFD:-

SF at B = - 150 KN

SF at D = -150 + 150 => 0

SF at C = -150 + 150 + 150 => 150 KN

SF at A = 150 KN

BMD:-

BM at B = 0

BM at D = 150 × 2 => 300 KN-m

BM at C => 150 × 4 - 150 × 2 => 300 KN-m

BM at A => 0

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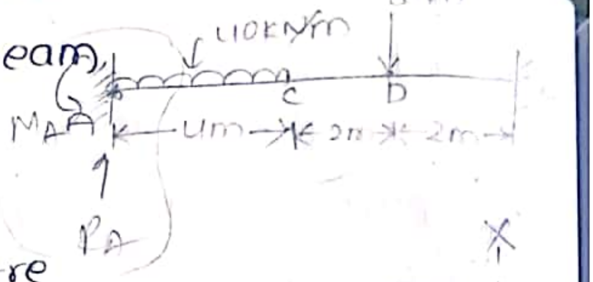
6/02/2020

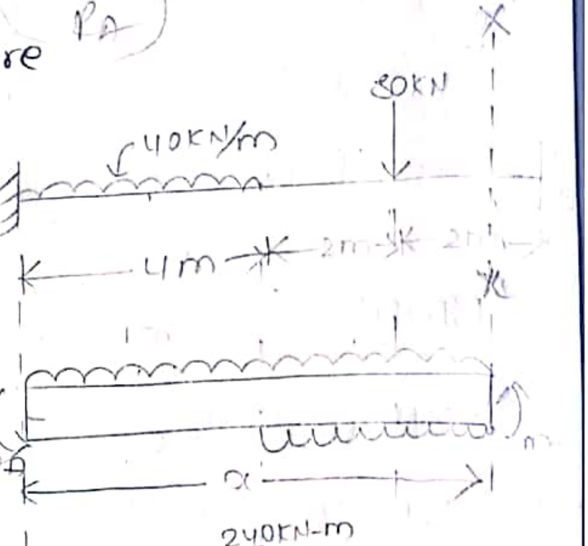
Example-2: For given beam,



(a) F.E.M

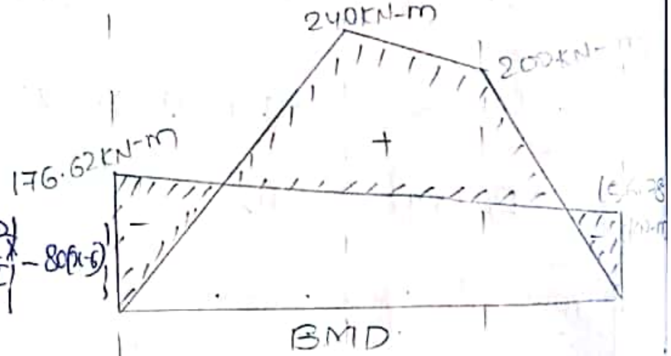
(b) Support

(c) Deflection at centre



$$M_x = R_A \times x - 80(x-6) - M_A + 40\frac{(x-4)(x-4)}{2} - 40 \times x \times \frac{x}{2}$$

$$M_x = R_A x - M_A - 40 \times x \times \frac{x}{2} - 80(x-6) + 40\frac{(x-4)(x-4)}{2}$$

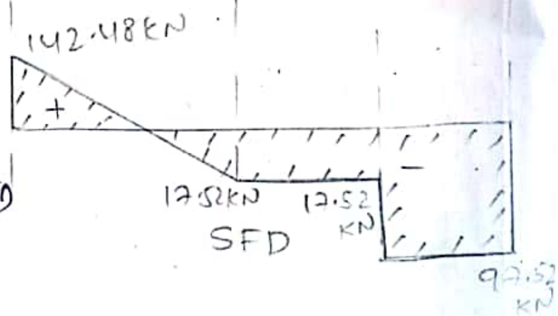


$$EI\frac{d^2y}{dx^2} = M_x$$

$$EI\frac{d^2y}{dx^2} = R_A x - M_A - \overset{20}{\cancel{40}} \times \frac{x^2}{\cancel{2}} - 80(x-6) + \overset{20}{\cancel{40}}\frac{(x-4)^2}{\cancel{2}}$$

Integrating on bothsides

$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} - M_A \cdot x - \frac{20 \times x^3}{3} + C_1 - \overset{40}{\cancel{80}}\frac{(x-6)^2}{\cancel{2}} + \frac{20(x-4)^3}{3} \rightarrow (1)$$



Integrating on bothsides

$$EI \cdot y = R_A \cdot \frac{x^3}{6} - M_A \cdot \frac{x^2}{2} - \frac{\overset{5}{\cancel{20}}x^4}{3 \times \cancel{4}} + C_1 x + C_2 - \frac{\overset{40}{\cancel{80}}(x-6)^3}{\cancel{2} \times 3} + \frac{\overset{5}{\cancel{20}}(x-4)^4}{3 \times \cancel{4}} \rightarrow (2)$$

Apply boundary conditions,

$\frac{dy}{dx} = 0,\ x = 0$ in equation (1), $\boxed{C_1 = 0}$

$y = 0,\ x = 0$ in eq (2), $\boxed{C_2 = 0}$

apply C_1, C_2 in eq (1) & (2)

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$$EI\frac{dy}{dx} = R_A\frac{x^2}{2} - M_A x - \frac{20x^3}{3} \Big| - 40(x-6)^2 \Big| + \frac{20}{3}(x-4)^3 \quad \longrightarrow (3)$$

$$EIy = R_A\frac{x^3}{6} - M_A\frac{x^2}{2} - \frac{5x^4}{3} \Big| - \frac{40(x-6)^3}{3} \Big| + \frac{5(x-4)^4}{3} \quad \longrightarrow (4)$$

substitude eq(3)&(4)

$x=8,\ \frac{dy}{dx}=0 \longrightarrow (3)$

$x=8,\ y=0 \longrightarrow (4)$

$$0 = R_A\frac{8^2}{2} - M_A\times 8 - \frac{20(8)^3}{3} \Big| - 40(8-6)^2 \Big| + \frac{20}{3}(8-4)^3$$

$$0 = 32R_A - 8M_A - 3146.66 \longrightarrow (5)$$

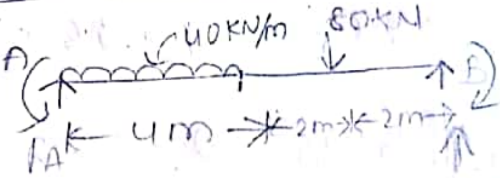
$$0 = R_A\frac{8^3}{6} - M_A\frac{8^2}{2} - \frac{5(8)^4}{3} \Big| - \frac{40(8-6)^3}{3} \Big| + \frac{5(8-4)^4}{3}$$

$$0 = 85.33R_A - 32M_A - 6506.66 \longrightarrow (6)$$

$$32R_A - 8M_A = 3146.66 \longrightarrow (5)$$

$$85.33R_A - 32M_A = 6506.66 \longrightarrow (6)$$

$\boxed{R_A = 142.48\ kN}$ $\boxed{M_A = 176.62\ kN\text{-}m}$



$\Sigma V = 0$

$R_A + R_B = 40\times4 + 80$

$R_A + R_B = 240$

$R_B = 240 - 142.48 \Rightarrow 97.52\ kN$

$\boxed{\therefore R_B = 97.52\ kN}$

$\Sigma M_A = 0$

$$+M_A + R_A(0) - 40(4)\left(\frac{4}{2}\right) - 80(6) + 8R_B + M_B = 0$$

$$+176.62 - 320 - 480 + (8\times 97.52) + M_B = 0$$

$\boxed{M_B = 156.78\ kN\text{-}m}$

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$x = 4$

$$EI\,y_{\substack{max\\centre}} = 142.48 \times \frac{4^3}{6} - 176.62\,\frac{(4)^2}{2} - \frac{5(4)}{3}$$

$EI = 15000\ kN/m^2$

$$y = \frac{-4799.94}{EI}$$

$$y = \frac{-319.84}{15000}$$

$$y = -0.02132\ m$$

$$\boxed{\therefore y = -21.32\ mm}$$

SFD :-

$SF_B = -97.52\ KN$

$SF_D = -97.52 + 80 \Rightarrow -17.52\ KN$

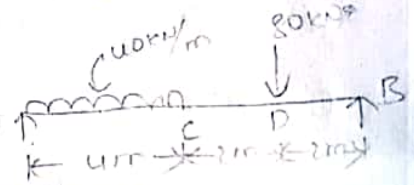
$SF_C = -17.52\ KN$

$SF_{A\ \text{due to udl}} = -17.52 + (40 \times 4) \Rightarrow 142.48\ KN$

BMD :-

$\Sigma V = 0$

$R_A + R_B = (40 \times 4) + 80$



$\Sigma M_A = 0$

$BM_B = 0$

$R_A(0) - 40 \times 4 \times \frac{4}{2} - 80(6) + 8R_B = 0$

$\boxed{\therefore R_B = 100\ KN}$ $\boxed{\therefore R_A = 140\ KN}$

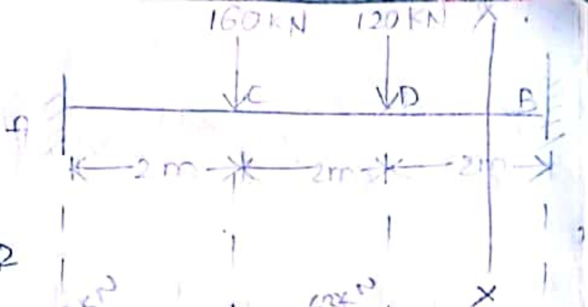
$BM_D = 100.00 \times 2 \Rightarrow 200\ KN\text{-}m$

$BM_C \Rightarrow 100.00 \times 4 - 80(2) \Rightarrow 240.00\ KN\text{-}m$

$BM_A = 0$

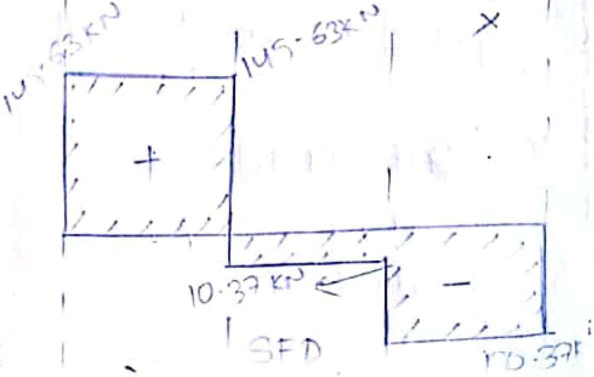
$BM_{\text{at 2m from A}} = 100.00 \times 6 - 80 \times 4 - 40 \times 2 \times \left[\frac{2}{2} + 2\right] \Rightarrow 40\ KN\text{-}m$

13/02/2020

1) 

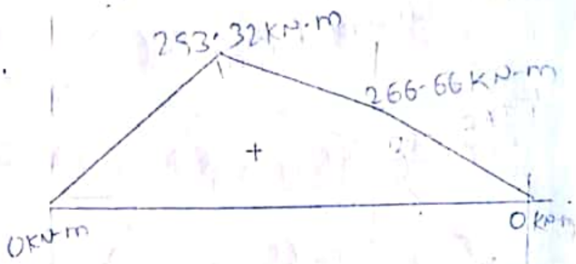
$M_x = -M_A + R_A \cdot x - 160\times 2 - 120\times 4 + 6R_B$

$M_x = R_A \times x - 120(x-4) - 160(x-2) - M_A$



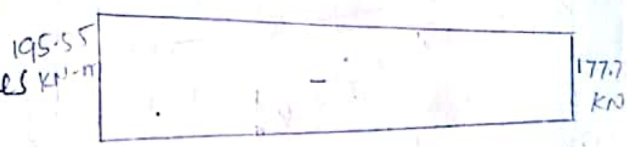
$M_x = R_A x - M_A - 160(x-2) - 120(x-4) \rightarrow (1)$

$\frac{d^2y}{dx^2} = -M_x \longrightarrow (2)$



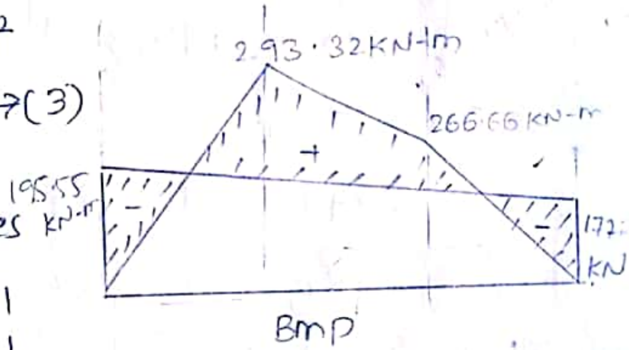
from (1) & (2)

$EI\frac{d^2y}{dx^2} = R_A x - M_A - 160(x-2) - 120(x-4)$



Integrate on bothsides

$EI\frac{dy}{dx} = R_A\frac{x^2}{2} - M_A x + C_1 - 160\frac{(x-2)^2}{2} - \frac{120(x-4)^2}{2} \rightarrow (3)$



Integrate on both sides

$EIy = R_A\frac{x^3}{2\times 3} - M\frac{x^2}{2} + C_1 x + C_2 - 160\frac{(x-2)^3}{2\times 3} - \frac{120(x-4)^3}{2\times 3} \rightarrow (4)$

apply boundary conditions in eq (3) & (4)

(1) $x=0,\ \frac{dy}{dx}=0$ $\quad \boxed{\therefore C_1 = 0}$

(2) $y=0,\ x=0$ $\quad \boxed{\therefore C_2 = 0}$

apply C_1, C_2 in eq (3) & (4)

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$$EI\frac{dy}{dx} = R_A\cdot\frac{x^2}{2} - M_A x - 160\frac{(x-2)^2}{2} - 120\frac{(x-4)^2}{2} \rightarrow (5)$$

$$EI\, y = R_A\cdot\frac{x^3}{6} - M_A\frac{x^2}{2} - \frac{160(x-2)^3}{6} - \frac{120(x-4)^3}{6} \rightarrow (6)$$

Substitude

$\frac{dy}{dx}=0,\ x=6$; $y=0,\ x=6$ in eq (5) & (6)

$$0 = R_A\cdot\frac{6^2}{2} - M_A(6) - 160\frac{(6-2)^2}{2} - 120\frac{(6-4)^2}{2}$$

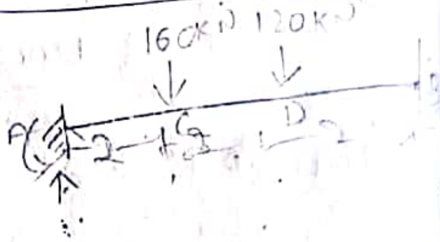
$$0 = 18R_A - 6M_A - 1520 \rightarrow (7)$$

$$0 = R_A\frac{(6)^3}{6} - M_A\frac{(6)^2}{2} - 160\frac{(6-2)^3}{6} - 120\frac{(6-4)^3}{6}$$

$$0 = 36R_A - 18M_A - 1866.66 \rightarrow (8)$$

$\boxed{\therefore R_A = 149.63\,KN}$, $\boxed{\therefore M_A = 195.55\,KN\text{-}m}$

From dia.



$R_A + R_B = 160+120$

$\boxed{\therefore R_B = 130.37\,KN}$

$\Sigma M_A = 0$

$R_A(0) - 160(2) - 120\times4 + 6R_B - M_B + M_A = 0$

$-800 + 6(130.37) - M_B + 195.55 = 0$

$-M_B + 177.77 = 0$

$\boxed{\therefore M_B = 177.77\,KN\text{-}m}$

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SFD:-

$SF_B = -130.37$ KN

$SF_D = -130.37 + 120 \Rightarrow -10.37$ KN

$SF_C \Rightarrow -130.37 + 120 + 160 \Rightarrow 149.63$ KN

$SF_A \Rightarrow 149.63$ KN

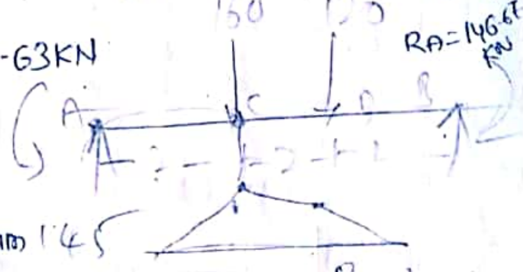
$2 \le x \le 4$

$\Sigma V = 0$: $R_A + R_B = 160 + 120 \Rightarrow 280$

$\Sigma M_A = 0$: $R_A(0) - 160(2) - 120(4) + 6R_B = 0$

$R_B = 133.33$ KN

$R_A = 146.67$ KN



BMD:- considered as S.S beam

$(R_A \times 2)$ $\quad R_A \times 4$ $\quad 16 \times 2$

$BM_B = \cancel{-177.77} \text{ KN-m} \; 0$

$BM_D = -177.77 + (130.37 \times 2) \Rightarrow 82.97$ KN-m

$BM_C = -177.77 + (130.37 \times 4) - (120 \times 2) \Rightarrow 103.71$ KN-m

$BM_A \Rightarrow -177.77 + (130.37 \times 6) - (120 \times 4) - (160 \times 2)$

$\Rightarrow -195.55$ KN-m

$BM_B = 0$

$BM_D = 133.33 \times 2 \Rightarrow 266.66$ KN-m $\quad BM_C \Rightarrow 133.33 \times 4 - 120(2)$

$\Rightarrow 293.32$ KN-m

$BM_A = 0$

when $\frac{dy}{dx} = 0$, there will be max. deflection in the span $\quad 2 \le x \le 4$

put $\frac{dy}{dx} = 0$ in eq (5)

$$0 = 149.63\frac{x^2}{2} - 195.55x - \cancel{160}^{80}\frac{(x-2)^2}{\cancel{2}}$$

$$0 = 74.815x^2 - 195.55x - 80(x^2 - 4x + 4)$$

$$0 = -5.185x^2 + 124.45x - 320$$

$$\boxed{\therefore x = 2.92\text{ m}}$$

Substu. in eq (6)

$$y = 149.63\frac{(2.92)^3}{6} - 195.55\frac{(2.92)^2}{2} - \frac{160(2.92-2)^3}{6}$$

$$y = -\frac{237.31}{EI} \text{ m}$$

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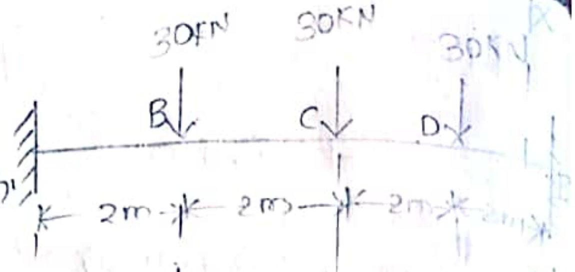
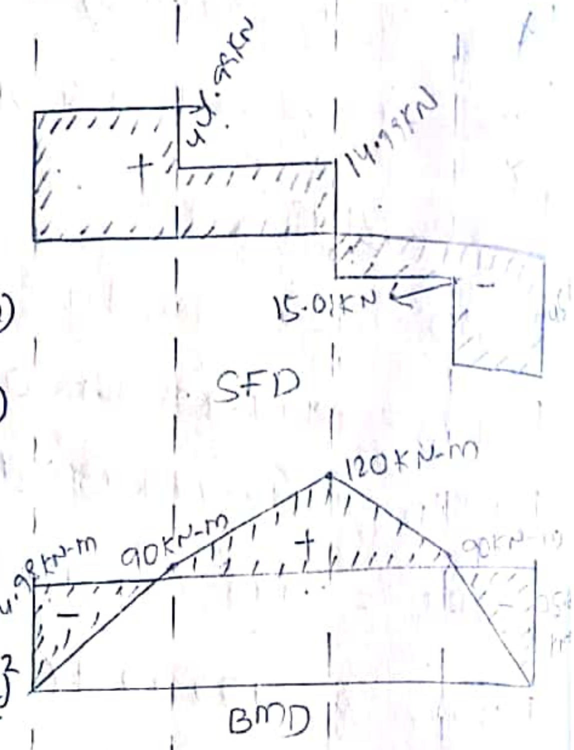
$$M_x = R_A \times x - m_A - 30(x-2) - 30(x-4) - 30(x-6) \rightarrow (1)$$



$$EI \frac{d^2y}{dx^2} = M \rightarrow (2)$$

From (1) & (2)

$$EI \frac{d^2y}{dx^2} = R_A x - m_A - 30(x-2) - 30(x-4) - 30(x-6)$$



Integrate on both sides, we get

$$EI \cdot \frac{dy}{dx} = R_A \cdot \frac{x^2}{2} - M_A x + c_1 - \frac{30(x-2)^2}{2} - \frac{30(x-4)^2}{2} - \frac{30(x-6)^2}{2} \rightarrow (3)$$

Integrate on both sides, we get

$$EI \cdot y = R_A \cdot \frac{x^3}{6} - M_A \cdot \frac{x^2}{2} + c_1 x + c_2 - \frac{30(x-2)^3}{6} - \frac{30(x-4)^3}{6} - \frac{30(x-6)^3}{6} \rightarrow (4)$$

Apply boundary conditions (3) & (4)

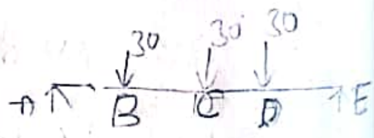
(1) $x=0, \frac{dy}{dx} = 0$

(2) $x=0, y=0$

We get $c_1 = 0, \quad c_2 = 0$

(3) $\frac{dy}{dx} = 0, x = 8$

(4) $y = 0, x = 8$



$\Sigma V = 0$

$R_A + R_E = 90$

$\Sigma M_A = 0$

$-30(2) - 30(4) - 30(6) + 8R_E = 0$

$\boxed{R_E = 45 \text{ KN}}$

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$$0 = R_A \frac{8^2}{2} - M_A(8) - 30\frac{(8-2)^2}{2} - 30\frac{(8-4)^2}{2} - 30\frac{(8-6)^2}{2}$$

$$32R_A - 8M_A - 840 = 0 \longrightarrow (5)$$

$$0 = R_A \frac{8^3}{6} - M_A \frac{8^2}{2} - 30\frac{(8-2)^3}{6} - 30\frac{(8-4)^3}{6} - 30\frac{(8-6)^3}{6}$$

$$85.33R_A - 32M_A - 1440 = 0 \longrightarrow (6)$$

solving (5) & (6) equations.

$\boxed{\therefore R_A = 44.99\ KN}$ $\boxed{\therefore M_A = 74.98\ KN\text{-}m}$

$\Sigma V_A = 0$

$R_A + R_B = 30 \times 3$

$\boxed{\therefore R_B = 45.01\ KN}$

$\Sigma M_A = 0$

$M_A + R_A(0) - 30(2) - 30(4) - 30(6) + 8R_B - M_B = 0$

$74.98 - 60 - 120 - 180 + 8(45.01) - M_B = 0$

$\boxed{\therefore M_B = 75.06\ KN\text{-}m.}$

$$EIy_{centre} = 44.99 \cdot \frac{(4)^3}{6} - 74.98 \cdot \frac{(4)^2}{2} - \frac{30(4-2)^3}{6}$$

$EI\,y_{centre} = -159.946\ m.$

SFD:-

SF at B = -45.01 KN

SF at D = $-45.01 + 30 \Rightarrow -15.01$ KN

SF at C = $-45.01 + 30 + 30 \Rightarrow 14.99$ KN

SF at B $\Rightarrow -45.01 + 30 + 30 + 30 \Rightarrow 44.99$ KN

SF at A = 44.99 KN

B.M.D:-

$BM_B = 0$

$BM_D = 45 \times 2 \Rightarrow 90$ KN-m

$BM_C \Rightarrow 45 \times 4 - 30 \times 2$
$\Rightarrow 120$ KN-m

$BM_B \Rightarrow 45 \times 6 - 30 \times 4 - 30 \times 2$
$\Rightarrow 90$ KN-m

$BM_A = 0$

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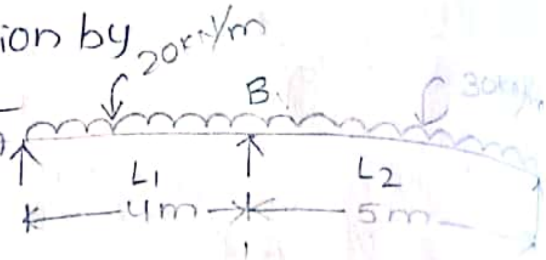
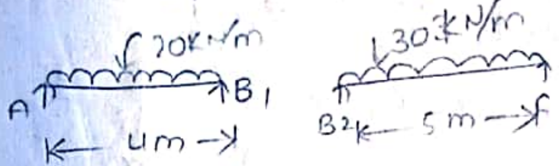
14/02/2020

$\Rightarrow$ Over hanging Beams:-

* Three moment equation by clapeyron's theorem:-

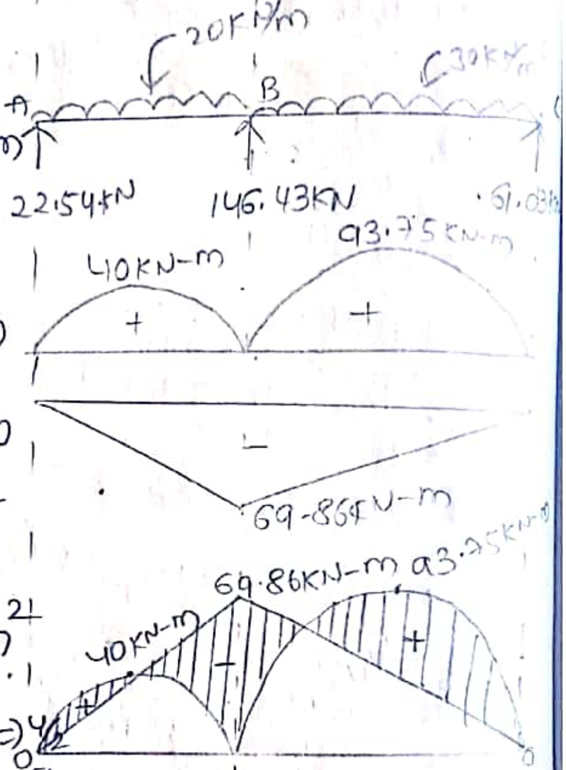


$$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 + \frac{6a_1x_1}{L_1} + \frac{6a_2x_2}{L_2} = 0 \rightarrow (1)$$



$$Bm_{AB} = \frac{20\times4\times\cancel{4}^2}{\cancel{8}} \Rightarrow \frac{20(4)^2}{8}$$
$$\Rightarrow 40KN\text{-}m$$

$$Bm_{BC} = \frac{\cancel{30}^{15}\times5\times5}{\cancel{2}} \Rightarrow \frac{30(5)^2}{8}$$
$$\Rightarrow 93.75KN\text{-}m$$



$$a_1 = \frac{2}{3}\times L_1\times h_{AB} \Rightarrow \frac{2}{3}\times4\times40$$
$$\Rightarrow 106.66m^2$$

$$a_2 = \frac{2}{3}\times5\times93.75 \Rightarrow 312.5m^2$$

$M_A = 0$, $M_C = 0$, $x_1 = L/2 \Rightarrow 4/2$

$x_2 = 5/2 \Rightarrow 5/2$

From (1),

$$0(L_1) + 2m_B(4+5) + 0(L_2) + \frac{6(106.66)}{4\times}\frac{4}{2} + \frac{6(312.5)(5)}{5\times 2}$$

$$18m_B = 1257.48$$

$$\boxed{\therefore m_B = -69.86\ KN\text{-}m}$$

$\Sigma V = 0$

$R_A + R_{B_1} = (20\times4)\ KN$

$R_A(0) - \frac{\cancel{20}^{10}\times4\times4}{\cancel{2}} + 4R_{B_1} = 0$

$\boxed{\therefore R_{B_1} = 40\ KN}$

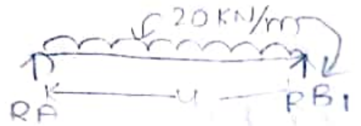
$\therefore \boxed{R_A = 40KN}$

$\therefore R_{B_2} = \frac{wl}{2} \Rightarrow \frac{30\times5}{2}$

$\Rightarrow 75KN$

$\boxed{\therefore R_C \Rightarrow 75KN}$

$\Sigma V = 0$



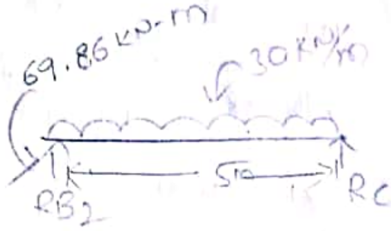
$R_A + R_{B1} = 20(4)$

$\Sigma M_A = 0$

$R_A(0) - 20(4)\times\frac{4}{2} + 4R_{B1} - 69.86$

$\boxed{\therefore R_{B1} = 57.465\,KN}$

$\boxed{R_A = 22.535\,KN}$



$\Sigma V = 0$

$R_{B2} + R_C = 30\times5$

$\Sigma M_A = 0$

$R_{B2}(0) - 30\times5\times\frac{5}{2} + 5R_C \cancel{+} 69.86 = 0$

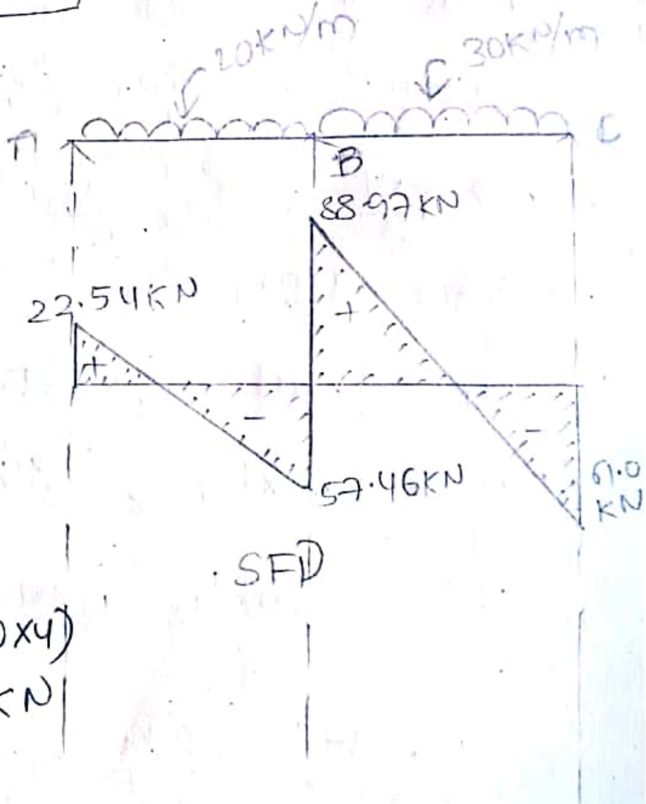
$R_C = \frac{305.14}{5}$ $\boxed{\therefore R_C = 61.028\,KN}$

$\boxed{\therefore R_{B2} = 88.98\,KN}$

$R_B = R_{B1} + R_{B2} \Rightarrow 57.465 + 88.98 \Rightarrow 146.44\,KN$

$\boxed{\therefore R_B = 146.44\,KN}$

SFD:-



SFD

SF at C $\Rightarrow -61.03\,KN$

SF at B due to u.d.l $\Rightarrow -61.03 + 30\times5$
$\Rightarrow 88.97\,KN$

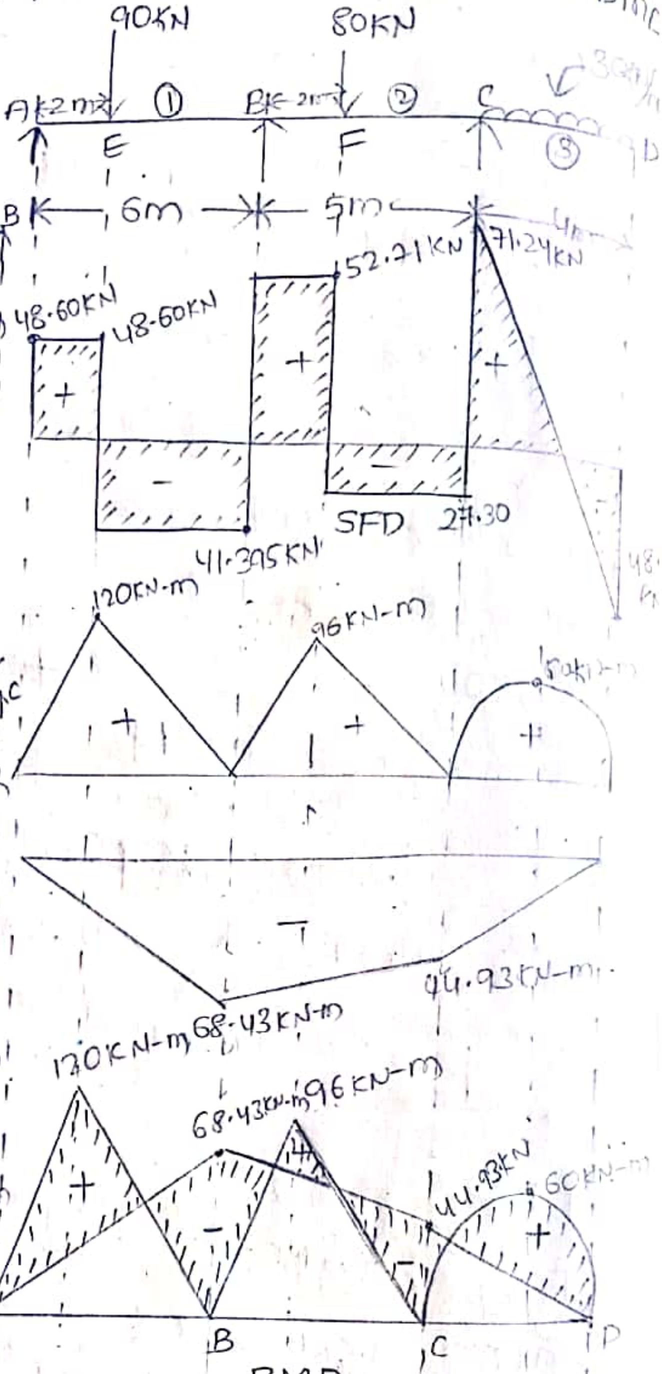
SF at B $\Rightarrow -61.03 + 30\times5$
$- 146.43$
$\Rightarrow -57.46\,KN$

SF at A due to u.d.l $\Rightarrow -57.46 + (20\times4)$
$\Rightarrow 22.54\,KN$

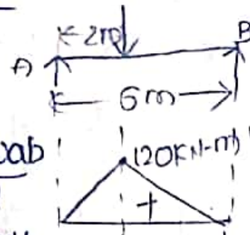
SF at A $\Rightarrow 22.54\,KN$

15/02/2020

1. Find R_B & R_C support reactions Draw BMD SFD.



Beam-I :- 90KN

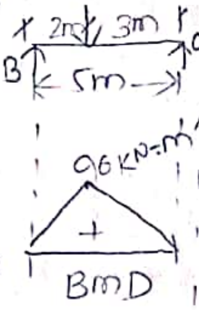


$BM = \frac{wab}{l}$

$\Rightarrow \frac{90 \times 2 \times 4}{6}$

$\Rightarrow 120 KN\text{-}m$

Beam-II :- 80KN

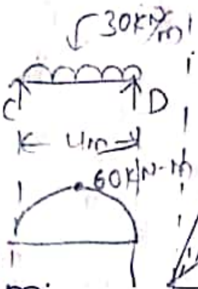


$BM = \frac{wab}{l}$

$\Rightarrow \frac{80 \times 2 \times 3}{5}$

$\Rightarrow 96 KN\text{-}m$

Beam-III



$BM = \frac{wl^2}{8}$

$\Rightarrow \frac{30 \times 4^2}{8}$

$\Rightarrow 60 KN\text{-}m$

$A_1 = \frac{1}{2} \times b \times h \Rightarrow \frac{1}{\cancel{2}} \times \overset{3}{\cancel{6}} \times 120 = 360 m^2$

$A_2 = \frac{1}{2} \times b \times h \Rightarrow \frac{1}{\cancel{2}} \times 5 \times \overset{48}{\cancel{96}} = 240 m^2$

$A_3 = \frac{2}{3} \times b \times h \Rightarrow \frac{2}{\cancel{3}} \times 4 \times \overset{20}{\cancel{60}} \Rightarrow 160 m^2$

$\bar{x}_1 = \frac{a+L}{3} \Rightarrow \frac{2+6}{3} \Rightarrow 2.67 m$
from Left

$\bar{x}_1 = \frac{b+L}{3} \Rightarrow \frac{4+6}{3} \Rightarrow 3.33 m$
from Right

$\bar{x}_{L2} = \frac{a+L}{3} \Rightarrow \frac{2+5}{3} \Rightarrow 2.33\,m$

$\bar{x}_{R2} = \frac{b+L}{3} = \frac{3+5}{3} \Rightarrow 2.66\,m$

$\bar{x}_{L3} = \bar{x}_{R3} = \frac{L}{2} \Rightarrow \frac{4}{2} \Rightarrow 2\,m$ [∵ because (symmetry)]

Clapeyron's theorem :-

Span-I :-

$M_A L_1 + 2M_B(L_1+L_2) + M_C L_2 + \frac{6a_1\bar{x}_1}{L_1} + \frac{6a_2\bar{x}_2}{L_2} = 0$

$(0)L_1 + 2M_B(6+5) + 5(M_C) + \frac{6(360)(2.67)}{6} + \frac{6(240)(2.67)}{5} = 0$

$22M_B = -1727.28$

$\boxed{\therefore M_B = -78.51\ KN\text{-}m}$

$22M_B + 5M_C = -1730.16 \longrightarrow (1)$

Span-II :-

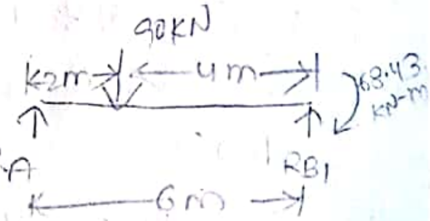
$M_B L_2 + 2M_C(L_2+L_3) + M_D(L_3) + \frac{6a_2\bar{x}_2}{L_2} + \frac{6a_3\bar{x}_3}{L_3} = 0$

$5M_B + 2M_C(5+4) + 0(L_3) + \frac{6(2.33)(240)}{5} + \frac{6(160)(2)}{4} = 0$

$5M_B + 18M_C = -1151.04 \longrightarrow (2)$

Solving eq (1) & (2)

$\boxed{\therefore M_B = -68.43\ KN\text{-}m}$ $\boxed{\therefore M_C = -44.93\ KN\text{-}m}$



$\Sigma V = 0$

$R_A + R_{B1} = 90$

$\Sigma M_A = 0$

$R_A(0) - 90(2) + 6(R_{B1}) - 68.43 = 0$

$\boxed{\therefore R_{B1} = 41.405\ KN}$

$\boxed{\therefore R_A = 48.595\ KN}$

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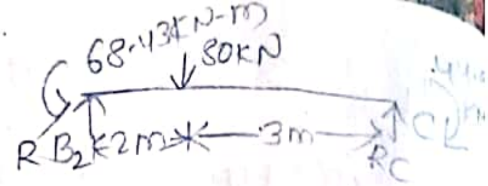
$\Sigma V = 0$



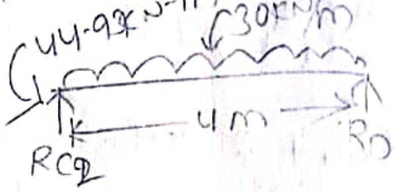
$R_{B_2} + R_C = 80$

$\Sigma M_A = 0$

$R_{B_2}(0) - 80(2) + 5R_C + 68.43 - 44.93 = 0$

$\boxed{\therefore R_C = 18.314 \text{ kN}}$ $\boxed{\therefore R_{C_1} = 27.3 \text{ kN}}$

$\boxed{\therefore R_{B_2} = 61.686 \text{ kN}}$ $\boxed{\therefore R_{B_2} = 52.7 \text{ kN}}$



$\Sigma V = 0$

$R_{C_2} + R_B = (30 \times 4)$

$\Sigma M_A = 0$

$R_{C_2}(0) - 30(4) \times \frac{4}{2} + 4R_D + 44.93 = 0$

$\boxed{\therefore R_D = 48.76 \text{ kN}}$

$\boxed{\therefore R_{C_2} = 71.23 \text{ kN}}$

$R_A = 48.595 \text{ kN} \simeq 48.6 \text{ kN}.$

$R_B = R_{B_1} + R_{B_2} = 41.405 + 52.7 \Rightarrow 94.105 \text{ kN}$

$R_C = R_{C_1} + R_{C_2} \Rightarrow 27.3 + 71.23 \Rightarrow 98.53 \text{ kN}$

$R_D = 48.76 \text{ kN}$

SFD:-

SF at D $= -48.76$ kN

SF at C due to udl $= -48.76 + 30 \times 4 \Rightarrow 71.24$ kN

SF at C $\Rightarrow -48.76 + 120 - 98.53 \Rightarrow -27.29$ kN

SF at F $\Rightarrow -48.76 + 120 - 98.53 + 80 \Rightarrow 52.71$ kN

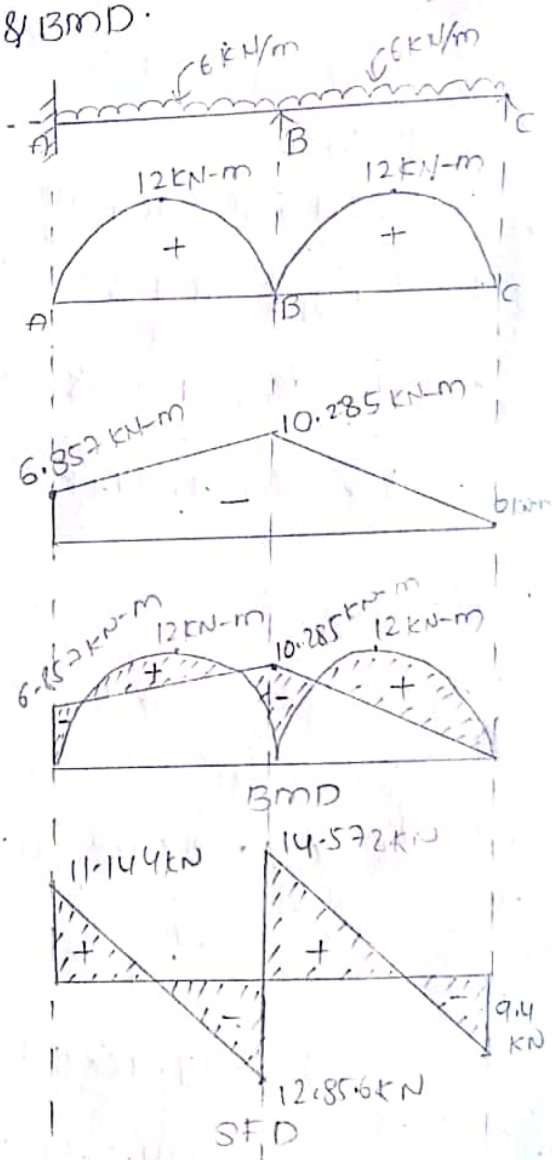
SF at B = 52.71 ~~+~~ 94.105 ⇒ −41.395 KN

SF at E ⇒ ~~94~~ −41.395 + 90 ⇒ 48.6 KN

SF at A ⇒ 48.6 KN.

17/02/2020

An continuous beam of span ABC, each span having equal length of 4m and carring udl of 6KN/m throught the beam. The support is fixed at "A", S.S. at C. Draw SFD & BMD.



$M_{A_1}(L_1) + 2M_A(L_1+L_2) + M_B L_2 + \frac{6a_0\bar{x}_0}{L_0} + \frac{6a_1\bar{x}_1}{L_1} = 0$

$M_{A_1}(0) + 2M_A(0+4) + 4M_B + \frac{6a_0\bar{x}_0}{L_0} + \frac{6a_1\bar{x}_1}{L_1} \rightarrow (1)$

$a_0 = 0$

$BM_0 = 0$

$BM_1 = \frac{wl^2}{8} = \frac{6\times4^2}{8} = 12\,KN\text{-}m$

$BM_2 = \frac{wl^2}{8} = 12\,KN\text{-}m$

$a_1 = \frac{2}{3}\times4\times12 \Rightarrow 32m^2$

$a_2 = \frac{2}{3}\times4\times12 \Rightarrow 32m^2$

$\bar{x}_1 = l/2 \Rightarrow 4/2 = 2m$

$\bar{x}_2 = 4/2 = 2m$

$\bar{x}_0 = 0$

From (1) & above values

$8M_A + 4M_B + 6(0) + \frac{6(32)(2)}{4} = 0$

$8M_A + 4M_B + 96 = 0 \longrightarrow (2)$

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$M_A L_1 + 2M_B(L_1+L_2) + M_C(L_2) + \frac{6a_1\bar{x}_1}{L} + \frac{6a_2\bar{x}_2}{L_2} = 0$

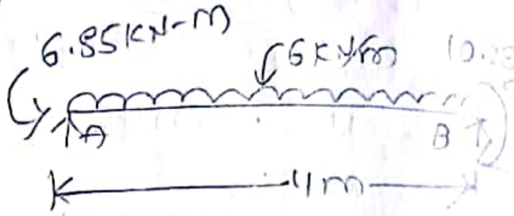
$M_A(4) + 2M_B(4+4) + 4(0) + \frac{6\times32\times2}{4} + \frac{6\times32\times2}{4} = 0$

$16M_B + 4M_A + 192 = 0 \longrightarrow (3)$

From eq (3) & (2)

$\boxed{\therefore M_A = -6.857 \text{ KN-m}}$

$\boxed{\therefore M_B = -10.285 \text{ KN-m}}$



$\Sigma V = 0$

$R_A + R_{B_1} = 6\times4 \text{ KN}$

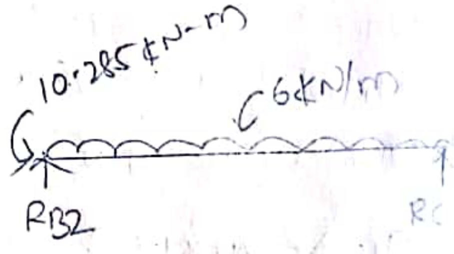
$\Sigma M_A = 0$

$R_A(0) + 6.85 - 6(4)\times\frac{4}{2} - 10.285 + 4R_{B_1} = 0$

$R_{B_1} = \frac{51.435}{4}$

$\boxed{\therefore R_{B_1} = 12.858 \text{ KN}}$

$\boxed{\therefore R_A = 11.142 \text{ KN}}$



$\Sigma V = 0$

$R_{B2} + R_C = 6\times4$

$\Sigma M_A = 0$

$R_{B_2}(0) - 6\times4\times\frac{4}{2} + 4R_C + 10.285 = 0$

$R_C = \frac{37.715}{4}$

$\boxed{\therefore R_C = 9.428 \text{ KN}}$

$\boxed{\therefore R_{B2} = 14.57 \text{ KN}}$

$R_B = R_{B_1} + R_{B_2} \Rightarrow 12.858 + 14.57 \Rightarrow 27.428 \text{ KN}$

$\boxed{\therefore R_B = 27.428 \text{ KN}}$

$\boxed{\therefore R_C = 9.428 \text{ KN}}$

Scanned with CamScanner

SFD:-

SF at C = -9.428 KN

SF at B due to udl = -9.428 + (6×4) ⇒ 14.572 KN

SF at B due to point load = -9.428 + 24 - 27.428 ⇒ -12.856 KN

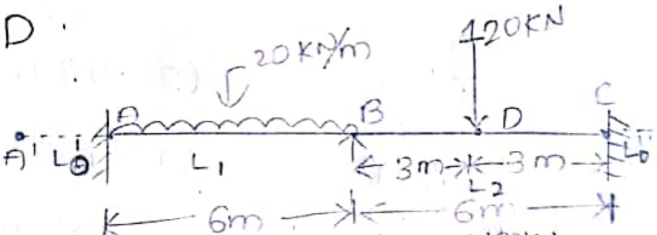
SF at A due to udl = -12.856 + 6×4 ⇒ 11.144 KN

18/02/2020

A continuous Beam span ABC of uniform section. Span AB and BC having length of 6m, End A and C are fixed support with support "B" shown in fig.

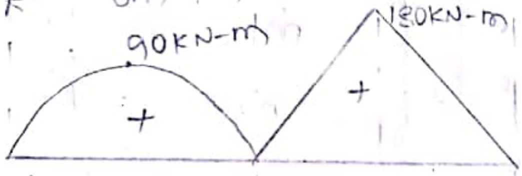
(1) Find support moment and reactions.

(2) Draw SFD & BMD.



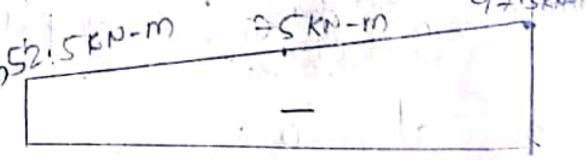
Span A'AB

$M_{A'} L_0 + 2M_A(L_0+L_1) + M_B(L_1) + \frac{6a_0\bar{x}_0}{L_0} + \frac{6a_1\bar{x}_1}{L_1} = 0 \rightarrow (1)$



$BM_{AA'} = 0$

$BM_{AB} = \frac{wl^2}{8} \Rightarrow \frac{20(6)^2}{8} \Rightarrow 90\,KN\text{-}m$



$BM_{BC} = \frac{wl}{4} \Rightarrow \frac{120(6)}{4} \Rightarrow 180\,KN\text{-}m$

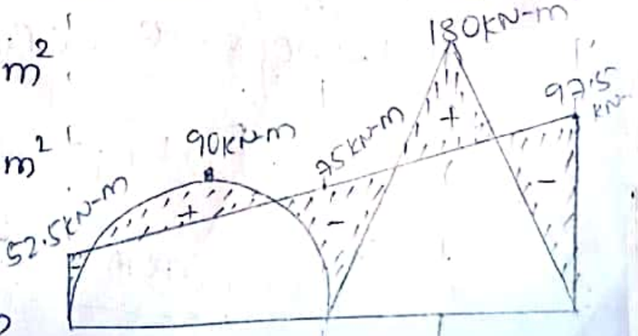
$BM_{CC'} = 0$

$a_0 = 0$

$a_1 = \frac{2}{3} \times 90 \times 6 \Rightarrow 360\,m^2$

$a_2 = \frac{1}{2} \times 6 \times 180 \Rightarrow 540\,m^2$

$a_{cc'} = 0$



$\bar{x}_0 = 0$, $\bar{x}_1 = \frac{6}{2} \Rightarrow 3m$

$\bar{x}_2 = 6/2 \Rightarrow 3m$, $\bar{x}_3 = 0$

BMD

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$$M_A(0)+2M_A(0+6)+M_B(6)+\frac{6(0)\bar{x}_0}{L_0}+\frac{6(360)(3)}{6}$$

$$12M_A+6M_B=-1080 \longrightarrow (2)$$

Span ABC :-

$$M_A L_1+2M_B(L_1+L_2)+M_C(L_2)+\frac{6a_1\bar{x}_1}{L_1}+\frac{6a_2\bar{x}_2}{L_2}=0$$

$$6M_A+2M_B(6+6)+6M_C+\frac{6(360)(3)}{6}+\frac{6(540)(3)}{6}=0$$

$$6M_A+24M_B+6M_C=-2700 \longrightarrow (3)$$

Span BCC' :-

$$M_B L_2+2M_C(L_2+L_3)+M_{C'}(L_0'')+\frac{6a_2\bar{x}_2}{L_2}+\frac{6a_3\bar{x}_3}{L_3}=0$$

$$M_B(6)+2M_C(6+0)+M_{C'}(0)+\frac{6(540)(3)}{6}+6(0)=0$$

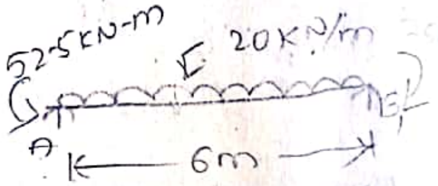
$$6M_B+12M_C=-1620 \longrightarrow (4)$$

solving eq (2) & (3), (4)

$$\therefore M_A=-52.5\ \text{kN-m}$$

$$\therefore M_B=-75\ \text{kN-m}$$

$$\therefore M_C=-97.5\ \text{kN-m}$$



$\Sigma V=0$

$$R_A+R_{B_1}=(20\times 6)\ \text{kN}$$

$\Sigma M_A=0$

$$R_A(0)-20(6)\times\frac{\cancel{6}^{3}}{\cancel{2}}+6R_{B_1}+52.5-75=0$$

$$\boxed{\therefore R_{B_1}=63.75\ \text{kN}}$$

$$\boxed{\therefore R_A=56.25\ \text{kN}}$$

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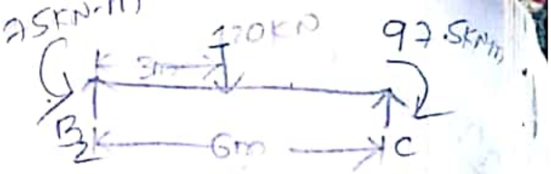
$\Sigma V = 0$

$R_A + R_B = 120$

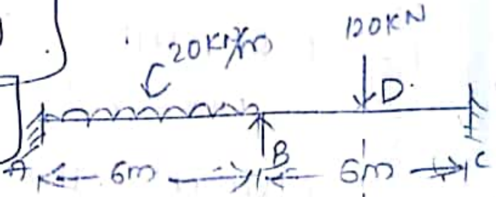


$\Sigma M_{B_2} = 0$

$R_{B2}(0) - 120(3) + 6R_C + 75 - 97.5 = 0$

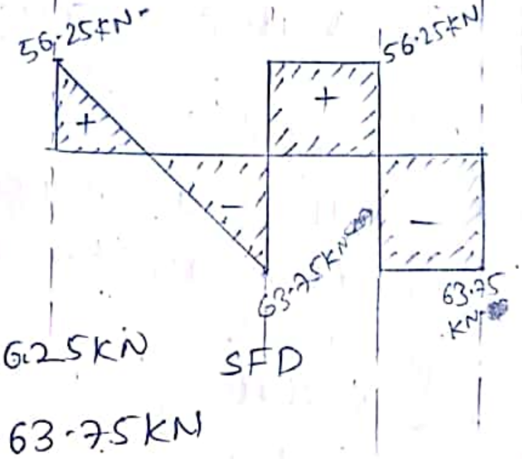
$\boxed{\therefore R_C = 63.75\,KN}$

$\boxed{\therefore R_{B_2} = 56.25\,KN}$



$R_{B1} + R_{B2} = R_B = 120\,KN.$

SFD:-



SF at C = $-63.75\,KN$

SF at D = $-63.75 + 120 \Rightarrow 56.25\,KN$

SF at B = $56.25 - 120 \Rightarrow -63.75\,KN$

SF at A due to udl $\Rightarrow -63.75 + 20(6) \Rightarrow 56.25\,KN$

SF at A = $56.25\,KN.$

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Direct and Bending Stresses

Syllabus

Stresses under the combined action of direct loading and bending moment, core of a section - determination of stresses in the case of chimneys, retaining walls and dams - condition for stability - stresses due to direct loading and bending moment about both axis.

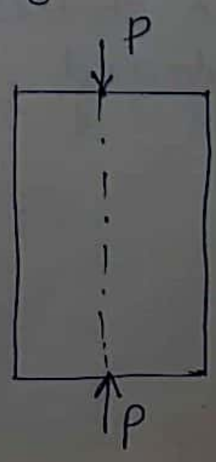
Introduction :-

Direct stress alone is produced in a body when it is subjected to an axial tensile or compressive load. and bending stress is produced in the body, when it is subjected to a bending moment. But if a body is subjected to axial loads and also bending moments, then both the stresses (i.e., direct & bending stresses) will be produced in the body. In this chapter, we shall study the Important cases of the members subjected to the direct & bending stresses. Both these stresses act normal to a c/s, hence the two stresses may be algebraically added into a single resultant stress.

Combined Bending and Direct stresses :-

Consider the case of a column subjected to a compressive load P acting along the axis of the column as shown in fig @.

This load will cause a direct compressive stress whose intensity will be uniform across the c/s of the column



let σ_0 = Intensity of the stress

A = Area of c/s

P = load acting on the column

Then stress,

$$\boxed{\sigma_0 = \frac{\text{load}}{\text{area}} = P/A}$$

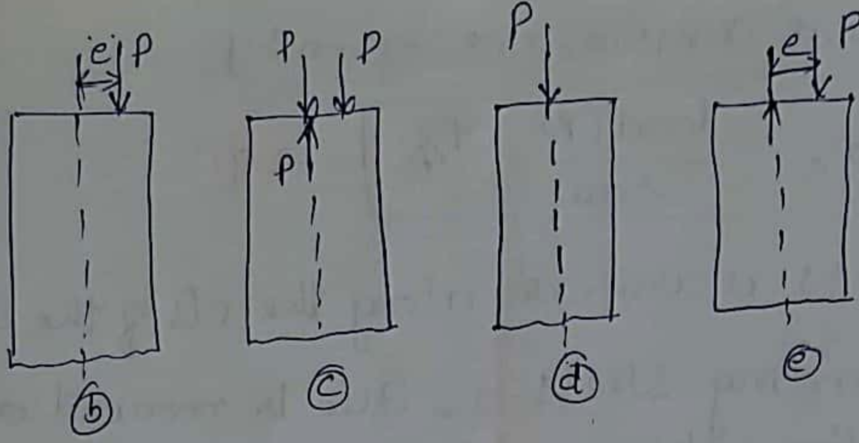
Now consider the case of a column subjected by a compressive load 'p' whose line of action is at a distance of 'e' from the axis of the column as shown in fig (b). Here 'e' is known as eccentricity of the load. The eccentricity load shown in fig (b) will cause direct stress and bending stress.

1. In fig (c), we have applied, along the axis of the column, two equal and opposite forces P. thus three forces are acting now on the column. one of the forces is shown in fig (d) and the other two forces are shown in (e).

2. The force shown in fig (d) is acting along the axis of the column and hence this force will produce a direct stress.

3. The forces shown in fig (e) will form a couple, whose moment will be P×e. This couple will produce a bending stress.

Hence an eccentric load will produce a direct stress as a bending stress. By adding these two stresses algebraically, a single stress can be obtained.



Resultant stress when a column of Rectangular section is subjected to an eccentric load

A column of rectangular section subjected to an eccentric load is shown in fig. let the load is eccentric with respect to the axis Y-Y as shown in fig (b). It is mentioned that an eccentric load causes direct as well as bending stress. Let us calculate these stresses.

Let P = Eccentric load on column

e = Eccentricity of the load

σ_0 = direct stress

σ_b = Bending stress

b = width of column

d = Depth of column

$\therefore$ Area of column section, $A = b \times d$

Now moment due to eccentric load of 'P' is given by,

$$M = \text{load} \times \text{eccentricity} = P \times e.$$

The direct stress (σ_0) is given by

$$\boxed{\sigma_0 = \frac{\text{load}(P)}{\text{Area}} = P/A} \longrightarrow (1)$$

This stress is uniform along the c/s of the column.

The bending stress σ_b due to moment at any point of the column section at a distance 'y' from the neutral axis Y–Y is given by

$$\frac{M}{I} = \frac{\sigma_b}{\pm y}$$

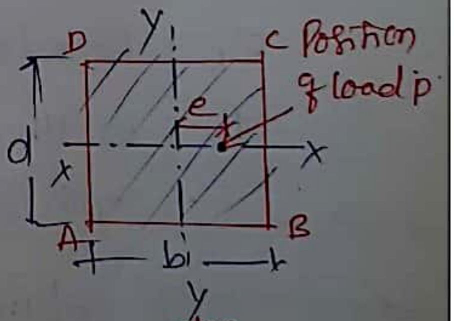
$$\therefore \boxed{\sigma_b = \pm \frac{M}{I} \times y} \longrightarrow (2)$$

elevation (a)

where I = Moment of Inertia of the column section about the neutral axis Y–Y $= \frac{db^3}{12}$

Substitute in eqn (2), we get

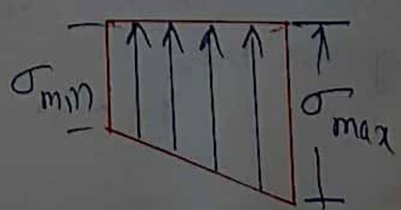
$$\boxed{\sigma_b = \pm \frac{M}{\frac{db^3}{12}} \times y = \frac{12M}{db^3} \times y}$$



plan (b)

The bending stress depends upon the value of 'y' from the axis Y–Y.

The bending stress at the extreme is obtained by substituting $y = b/2$ in the above value.



(c)

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$$\therefore \sigma_b = \pm \frac{\cancel{12}6M}{d b^{\cancel{3}}} \times \frac{\cancel{b}}{\cancel{2}} , = \pm \frac{6M}{d \cdot b^2}$$

$$\because (M = P \times e)$$

$$= \pm \frac{6P \times e}{d b^2}$$

$$= \pm \frac{6P \times e}{d \cdot b \cdot b} = \pm \frac{6P \times e}{A \cdot b}$$

$$(\because \text{Area} = b \times d = A)$$

The resultant stress at any point will be the algebraic sum of direct stress & bending stress.

If y is taken positive on the same side of y-y as the load, then bending stress will be of the same type as the direct stress. Here direct stress is compressive and hence bending stress will also be compressive towards the right of the axis Y-y. Similarly bending stress will be tensile towards the left of the axis y-y. Taking compressive stress as positive and tensile stress as negative we can find the maximum and minimum stress at the extremities of the section. The stress will be maximum along the layer BC and minimum along layer AD.

Let σ_{max} = maximum stress (i.e. stress along BC)

σ_{min} = minimum stress (i.e, stress along AD)

Then σ_{max} = Direct stress + Bending stress

$$= P/A + \frac{6P \cdot e}{Ab}$$

$$\boxed{\sigma_{max} = P/A \left(1 + \frac{6 \times e}{b}\right)} \longrightarrow (A)$$

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and σ_{min} = Direct stress - Bending stress

$$= \sigma_0 - \sigma_b$$

$$= P/A - \frac{6P.e}{A.b} = P/A\left(1 - \frac{6\times e}{b}\right)$$

$$\boxed{\sigma_{min} = P/A\left(1 - \frac{6\times e}{b}\right)} \rightarrow (B)$$

These stresses are shown in fig (1) (C). The resultant stress along the width of the column will be vary by a straight line law.

If σ_{min} is -ve then the stress along the layer AD will be tensile. If σ_{min} is zero then there will be no tensile stress along the width of the column. If σ_{min} is +ve then the there will be only compressive stress along the width of the column.

Problems :-

A Rectangular column of width 200mm and of thickness 150mm carries a point of 240kN at an eccentricity of 10mm. Determine the maximum and minimum stresses on the section.

Sol^n.

$b = 200mm$

$d = 150mm$

$A = b\times d$

$= 200\times 150 = 30000mm^2$

Eccentric load

$P = 240kN$

$= 240\times 10^3 N$

$e = 10mm$

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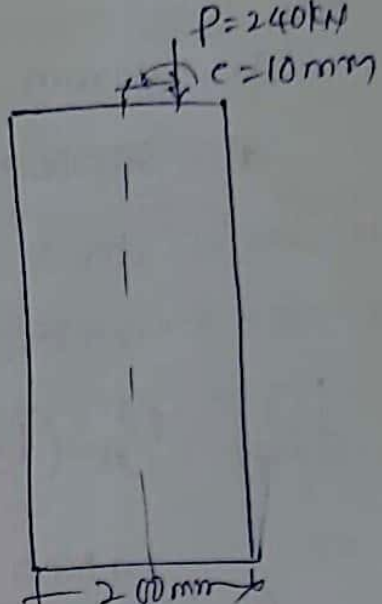
let σ_{max} = Maximum stress

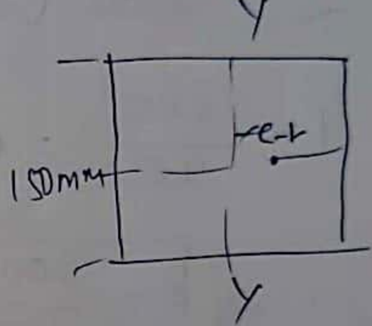
σ_{min} = minimum stress



$$\sigma_{max} = \frac{P}{A}\left(1+\frac{6\times e}{b}\right)$$

$$= \frac{240\times10^3}{30000}\left(1+\frac{6\times10}{200}\right)$$

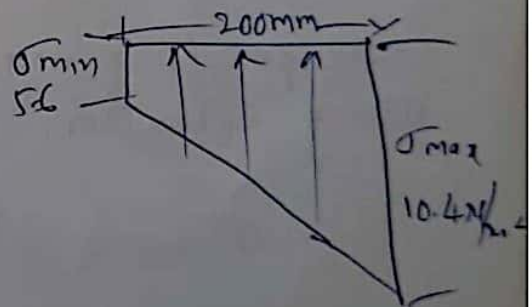
$$= 8(1+0.3) = 10.4\ N/mm^2$$



$$\sigma_{min} = \frac{P}{A}\left(1-\frac{6\times e}{b}\right)$$

$$= \frac{240\times10^3}{30000}\left(1-\frac{6\times10}{200}\right)$$

$$= 8(1-0.3) = 5.6\ N/mm^2$$



(2) A Rectangular column width 200mm and of thickness 150mm. ~~carries a point load of 240KN at an~~ The minimum stress on the section is zero then find the eccentricity of the point load of 240KN acting on the rectangular column. also calculate the corresponding maximum stress on the section.

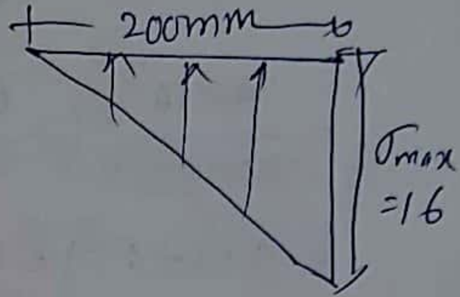
Sol^n:

$b = 200\,mm,\quad d = 150\,mm,\quad P = 240000\,N$

$A = 30000\,mm^2$

Minimum stress.

Let e = eccentricity



$$\sigma_{min} = \frac{P}{A}\left(1 - \frac{6\times e}{b}\right)$$

$$0 = \frac{240000}{30000}\left(1 - \frac{6\times e}{200}\right)$$

$$\text{or } 1 - \frac{6\times e}{200} = 0 \quad \text{or } 1 = \frac{6\times e}{200}$$

$$\therefore \boxed{e = \frac{200}{6} = 33.33\,mm}$$

corresponding maximum stress is obtained by using equation

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{6\times e}{b}\right)$$

$$= \frac{240000}{30000}\left(1 + \frac{6\times 33.3}{200}\right)$$

$$= 16\,N/mm^2$$

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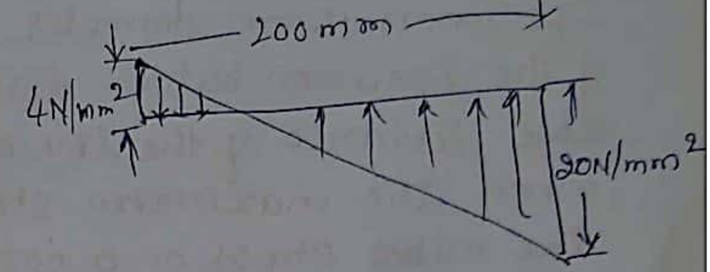
18/04/2020

③ A Rectangular column of width 200mm and of thickness 150mm carries a point load of 240KN at an eccentricity of 50mm then find the maximum and minimum stresses on the section. Also plot these stresses along the width of the section.

Solⁿ:-



Given data

$b = 200mm$

$d = 150mm$

$P = 240000N$

$A = b \times d = 200 \times 150 = 30000mm^2$

Eccentricity,

$e = 50mm.$

(i) Maximum stress (σ_{max}):-

$$\sigma_{max} = P/A\left(1+\frac{6e}{b}\right) = \frac{240000}{30000}\left(1+\frac{6\times50}{200}\right)$$

$$= 8(1+1.5) = 20N/mm^2.$$

(ii) Minimum stress (σ_{min}) is given by equation

$$\sigma_{min} = P/A\left(1-\frac{6e}{b}\right) = \frac{\cancel{240000}^{8}}{\cancel{30000}_{1}}\left(1-\frac{6\times50}{200}\right)$$

$$= 8(1-1.5) = -4N/mm^2$$

Negative sign means tensile stress.

(i) The minimum stress is zero when $e = \frac{200}{6}$ @ $\frac{b}{6}$ mm (as $b = 200mm$).

(ii) The minimum stress is +ve (i.e, compressive) when $e < b/6$. This clear Problem $e = 10mm$, less then $200/6$

(iii) The minimum stress is -ve (i.e. tensile) when $e > b/6$. This is clear from Problem. 3 in which $e = 50mm$. which is more then $b/6 = \frac{200}{6} = 33.33mm$.

(2) A line of thrust, in a compression testing specimen 15mm diameter, is parallel to the axis of the specimen but is displaced from it. Calculate the distance of the line of thrust from the axis when the maximum stress is 20% greater than the mean stress on a normal section.

Given data :-

$d = 15mm$

$\therefore$ Area $A = \pi/4 \times 15^2$

$$\boxed{A = 176.714 mm^2}$$

σ_{max} = 20% greater than mean

$= \frac{120}{100} \times$ mean stress

$$\boxed{\sigma_{max} = 1.2 \times \text{mean stress}}$$

P = compressive load on specimen

e = Eccentricity.

Mean stress $= \frac{\text{load}}{\text{area}} = \frac{P}{176.71}$ N/mm^2

we know the moment,

$$M = P \times e$$

$\therefore$ Bending stress is given by

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$\therefore \quad \sigma_b = \frac{M}{I} \times y$

$\therefore$ Maximum bending stress will be when $y = \pm d/2$

Hence maximum bending stress is given by.

$$\sigma_b = M/I \times (\pm d/2)$$

$$= \pm M/I \times d/2$$

$$= \pm \frac{M}{\frac{\pi d^4}{64}} \times d/2 \qquad \left(\because I = \frac{\pi d^4}{64}\right)$$

$$= \pm \frac{32M}{\pi d^3}$$

$$\boxed{\sigma_b = \pm \frac{32 P \times e}{\pi d^3}} \qquad (\because M = P \times e)$$

Direct stress due to load is given by.

$$\sigma_a = P/A = \frac{P}{176.714}$$

$\therefore$ Maximum stress = Direct stress + Bending stress

$$= \sigma_a + \sigma_b .$$

$$\sigma_{max} = \frac{P}{176.714} + \frac{32 P \times e}{\pi d^3} \longrightarrow (1)$$

But $\sigma_{max} = 1.2 \times$ mean stress

$$= 1.2 \frac{P}{176.714} \longrightarrow (2).$$

Equating (1) & (2)

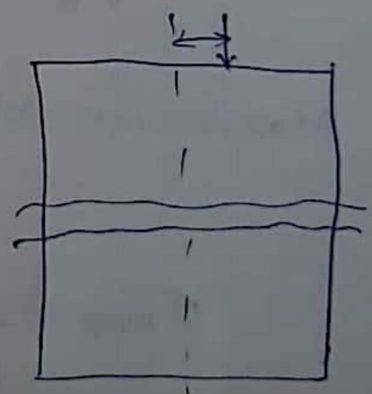
$$\frac{P}{176.714} + \frac{32P \times e}{\pi d^3} = 1.2 \times \frac{P}{176.714}$$

$$\frac{32P \times e}{\pi d^3} = \frac{1.2P}{176.714} - \frac{P}{176.714} = \frac{0.2P}{176.714}$$

$$\frac{32\cancel{P}\times e}{\pi d^3} = \frac{0.2\cancel{P}}{176.714} \quad \therefore \quad \frac{32e}{\pi d^3} = \frac{0.2}{176.714}$$

$$\therefore \quad e = \frac{0.2\pi d^3}{32 \times 176.714} = \frac{0.2 \times \pi \times 15^3}{32 \times 176.714} = \underline{\underline{0.375\text{ mm}}}$$

(5) A hollow rectangular column of external depth 1m and external width 0.8m is 10cm thick. Calculate the maximum and minimum stress in the section of the column if a vertical load of 200kN is acting with an eccentricity of 15cm as shown in fig.



<u>Given data</u>

$B = 0.8\text{m} = 800\text{mm}$

$D = 1.0\text{m} = 1000\text{ mm}$

Thickness of wall, $t = 10\text{cm} = 100\text{mm}$.

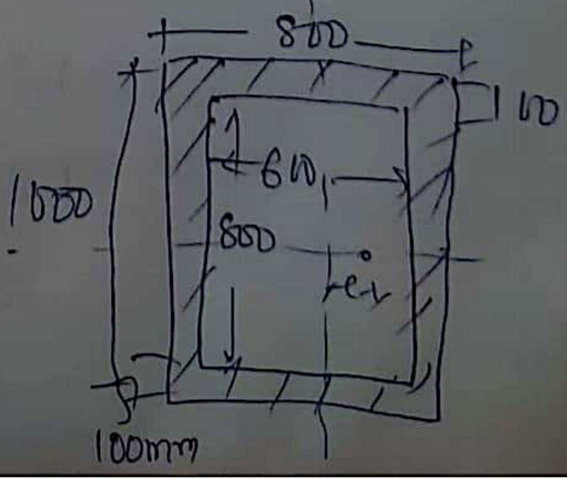
Inner width, $b = B - 2\times100$

$= \underline{600\text{mm}}$

Inner depth $= D - 2\times t$

$= 1000 - 2\times100$

$d = 800\text{ mm}$



$\therefore$ Area, $A = B \times D - b \times d$

$$= 800 \times 1000 - 600 \times 800$$

$$A = 320000 mm^2.$$

m.o.I about y-y axis is given by

$$I = \frac{DB^3}{12} - \frac{db^3}{12} = \frac{1000 \times 800^3}{12} - \frac{800 \times 600^3}{12}$$

$$= 42.66 \times 10^9 - 14.4 \times 10^9$$

$$I = 28.26 \times 10^9 mm^4$$

Eccentric load, $P = 200 kN = 200000 N$

Eccentricity, $e = 15 cm = 150 mm$

we know that the moment,

$$M = P \times e$$

$$= 200,000 \times 150$$

$$\boxed{M = 3000000 N\text{-}mm}$$

The bending stress is given by

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\therefore \sigma_b = \frac{M}{I} \times y$$

$$= \frac{\cancel{3000000}}{}$$

maximum bending stress will be when

$$y = \pm 400 mm$$

$$\therefore \sigma_b = \frac{M}{I} \times (\pm 400)$$

$$= \pm \frac{30000000}{28.26 \times 10^9} \times 400$$

$$= \pm 0.4246 \text{ N/mm}^2$$

Direct stress is given by

$$\sigma_d = P/A = \frac{200000}{320000}$$

$\therefore$ Maximum stress $= \sigma_d + \sigma_b = 0.625 + 0.4246$

$$\sigma_{max} = 1.0496 \text{ N/mm}^2 \text{ (Comp)}$$

$$\sigma_{min} = \sigma_d - \sigma_b$$

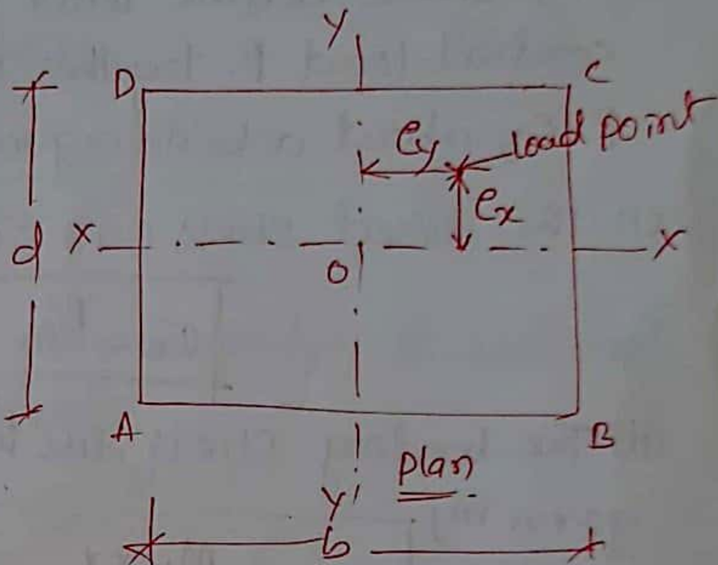
$$= 0.625 - 0.4246$$

$$= 0.2004 \text{ N/mm}^2 \text{ (Comp)}$$

Resultant stress when a column of Rectangular Section is Subjected to a load which is Eccentric to both axes.

A column of rectangular Section ABCD, Subjected to a load which is eccentric to both axis, is shown in fig.

let



plan

P = Eccentric load on column

e_x = Eccentric load ~~on column~~ about X-X axis

e_y = Eccentric load about Y-Y axis

b = width of column

d = depth of column

σ_d = direct stress

σ_{bx} = Bending stress due to eccentricity e_x

σ_{by} = Bending stress due to eccentricity e_y

M_x = Moment of load about X-X axis

$= P \times e_x$

M_y = Moment of load about Y-Y axis

$= P \times e_y$

I_{xx} = Moment of Inertia about x-x axis

$$= \frac{bd^3}{12}$$

I_{yy} = moment of Inertia about y-y axis

$$= \frac{db^3}{12}$$

Now the eccentric load is equivalent to a central load P, together with a bending moment $P \times e_y$ about a bending moment $P \times e_x$ about x-x axis

(i) The direct stress (σ_0) is given by

$$\boxed{\sigma_0 = P/A} \rightarrow (1)$$

(ii) The bending stress due to eccentricity e_y is given by

$$\boxed{\sigma_{by} = \frac{M_y \times x}{I_{yy}} = \frac{P \times e_y \times x}{I_{yy}}} \rightarrow (2)$$

In the above equation x varies from $-b/2$ to $+b/2$

(iii) The bending stress due to eccentricity e_x is given by,

$$\sigma_{bx} = \frac{M_x \times y}{I_{xx}} = \frac{P \times e_x \times y}{I_{xx}}$$

In the above equation, y varies from $-d/2$ to $+d/2$

The resultant stress at any point on the section

$$= \sigma_0 \pm \sigma_{by} \pm \sigma_{bx}$$

$$= P/A \pm \frac{M_y \times x}{I_{yy}} \pm \frac{M_x \times y}{I_{xx}}$$

(i) At the point C, the co-ordinates x & y are positive hence the resultant stress will be maximum.

(ii) At the point A, the co-ordinates x & y are -ve and hence the resultant stress will be minimum.

(iii) At the point B, x is +ve & y is -ve and hence resultant stress.

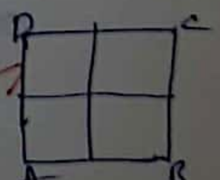
$$= \frac{P}{A} + \frac{M_y . x}{I_{yy}} - \frac{M_x . y}{I_{xx}}$$

(iv) At the point D, x is -ve and y is +ve and hence resultant stress.

$$= \frac{P}{A} - \frac{M_y . x}{I_{yy}} + \frac{M_x . y}{I_{xx}}$$

Problems :-

A column is rectangular in c/s of 300mm x 400mm in dimensions. The column carries an eccentric point load of 360KN on one diagonal at a distance of quarter diagonal length from a corner. Calculate the stresses at all four corners. Draw stress distribution diagrams for any adjacent sides.



Sol^n

b = 300mm, d = 400mm

(ii) Resultant stress at a point B

At point B, x = 150mm and y = -200mm.

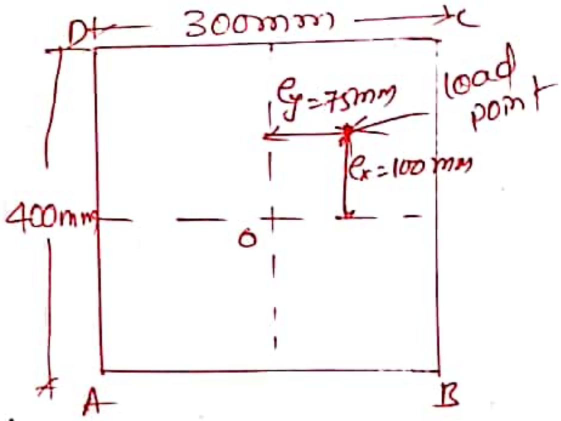
Resultant stress at B $= \frac{P}{A} + \frac{M_y \times 150}{I_{yy}} - \frac{M_x \times 200}{I_{xx}}$

$$= \cancel{\frac{360,000}{12\times10^4} + \frac{27000000}{9\times10^8} - \frac{36000000\times200}{16\times10^8}}$$

$$= \cancel{3+4.5-4.5}$$

$$= \cancel{3 N/mm^2 \text{ (Compression)}}$$

Given data



$b = 300mm$

$d = 400mm$

$\therefore$ Area, $A = b\times d = 300\times400$

$= 12\times10^4 mm^2$

Eccentric load, $P = 360kN = 360000N$

The eccentric load is acting at point E, where distance EC = one quarter of diagonal AC.

Now diagonal $AC = \sqrt{300^2+400^2} = 500mm$

$\therefore$ In $\triangle CAB$ $\tan\theta = \frac{4}{3} = \frac{opp}{adj}$

$\cos\theta = 3/5$ and $\sin\theta = 4/5$

$\cos\theta = \frac{adj}{hyp}$ and $\sin\theta = \frac{opp}{hyp}$

Also $OE = EC = 1/4$ of AC

$= 1/4 \times 500 = 125mm$

and $e_x = EF = OE\sin\theta = 125\times4/5 = 100mm$

$e_y = OF = OE\cos\theta = 125\times3/5 = 75mm$

moment of load about x-x axis.

$M_x = P\times e_x = 360000\times100 = 36\times10^6 N\text{-}mm$

moment of load about y-y axis

$M_y = P\times e_y = 360000\times75 = 27000000 N\text{-}mm$

Also $I_{xx} = \frac{1}{12} \times 300 \times 400^3 = 16 \times 10^8 mm^4$

$I_{yy} = \frac{1}{12} \times 400 \times 300^3 = 9 \times 10^8 mm^4$

The resultant stress at any point is given by equation as

$$\text{Resultant stress} = P/A + \frac{M_y \times x}{I_{yy}} + \frac{M_x \times y}{I_{xx}}$$

(i) <u>Resultant stress at point C.</u>

At point C, $x = 150mm$ & $y = 200mm$

∴ Resultant stress at C

$$= P/A + \frac{M_y \times 150}{I_{yy}} + \frac{M_x \times 200}{I_{xx}}$$

$$= \frac{360 \times 10^3}{12 \times 10^4} + \frac{270 \times 10^5}{9 \times 10^8} + \frac{360 \times 10^5 \times 200}{16 \times 10^8}$$

$= 3 + 4.5 + 4.5 \ N/mm^2$

$= 12 \ N/mm^2$ (compressive)

(ii) <u>Resultant stress at point B</u>

At point B, $x = 150mm$ & $y = -200mm$

$$\text{Resultant stress at point B} = P/A + \frac{M_y \times 150}{I_{yy}} + \frac{M_x \times (-200)}{16 \times 10^8}$$

$$= \frac{360 \times 10^3}{12 \times 10^4} + \frac{27 \times 10^6}{9 \times 10^8} - \frac{36 \times 10^6 \times 200}{16 \times 10^3}$$

$= 3 + 4.5 - 4.5$

$= 3 \ N/mm^2$ (Compressive).

(iii) Resultant stress at point A

A point A, $x = -150mm$ & $y = -200mm$

∴ Resultant stress at point A

$$= P/A + \frac{M_y \times (-150)}{I_{yy}} + \frac{M_x \times (-200)}{I_{xx}}$$

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$$= \frac{36\times10^4}{12\times10^4} - \frac{27\times10^6}{9\times10^8} - \frac{36\times10^6\times200}{16\times10^8}$$

$$= 3 - 4.5 - 4.5$$

$$= \underline{-6\ N/mm^2}\ (\text{Tensile})$$

(iv) Resultant stress at point D

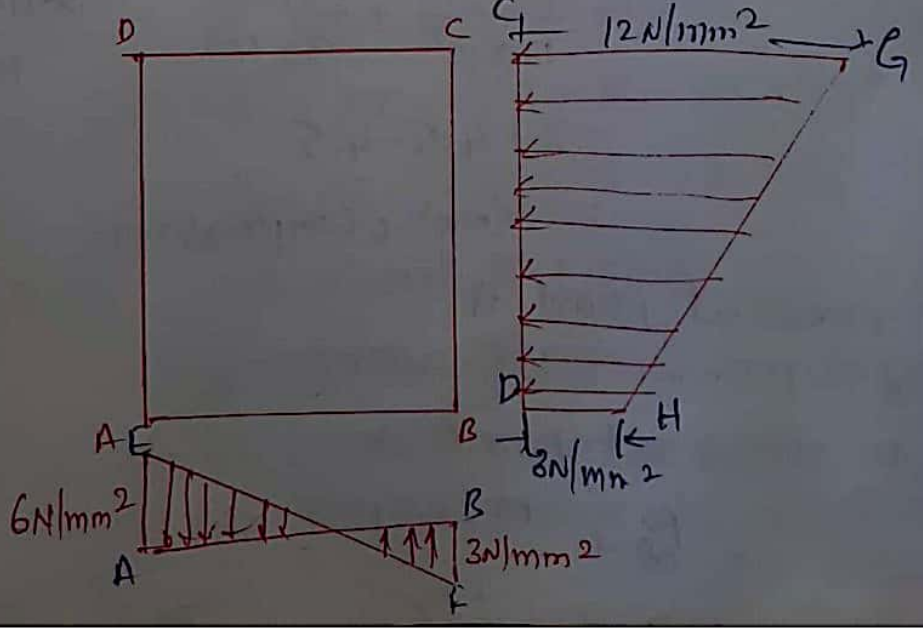
At point D, $x = -150mm$ & $y = 200mm$

$$\therefore = P/A + \frac{M_y(-150)}{I_{YY}} + \frac{M_x\times200}{I_{xx}}$$

$$= \frac{36\times10^4}{12\times10^4} - \frac{27\times10^6}{9\times10^8} + \frac{36\times10^6\times200}{16\times10^8}$$

$$= 3 - 4.5 + 4.5$$

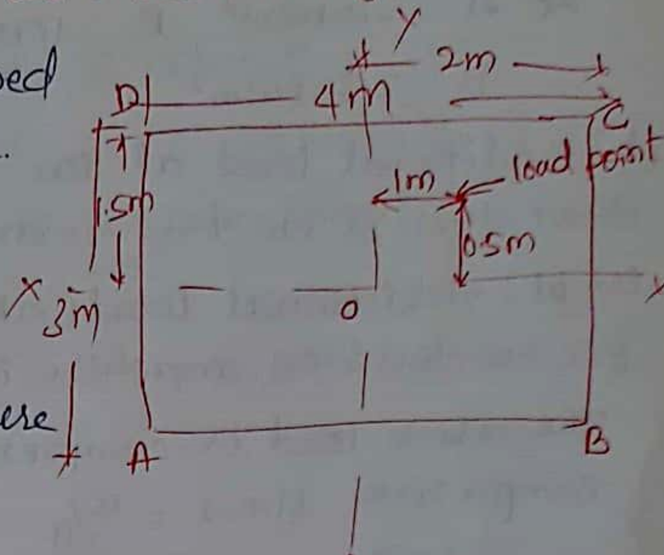
$$= \underline{3\ N/mm^2}\ (\text{Compressive})$$



② A masonry pier of 3m×4m supports a vertical load of 80kN as shown in fig.

ⓐ Find the stresses developed at each corner of the pier.

ⓑ what additional load should be placed at the centre of the pier, so that there is no tension anywhere in the pier section?



ⓒ What are the stresses at the ~~centre~~ corners with the additional load in the centre?

Soln. $b = 4m$, $d = 3m$ $\therefore A = 4 \times 3 = 12 m^2$

point load, $P = 80 kN$.

Eccentricity of load about x-x axis.

$$e_x = 0.5m$$

Eccentricity of load about Y-Y axis

$$e_y = 1.0m$$

moment of load about x-x axis

$$M_x = P \times e_x = 80 \times 0.5 = 40 kN\text{-}m$$

$$M_y = P \times e_y = 80 \times 1.0 = 80 kN\text{-}m$$

moment of Inertia about x-x axis,

$$I_{xx} = \frac{1}{12} \times 4 \times 3^3 = \underline{9 m^4}$$

Similarly, $I_{YY} = \frac{1}{12} \times 3 \times 4^3 = \underline{16 m^4}$

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(a) Resultant stresses

at $A = -10\ kN/m^2$, $B = 10\ kN/m^2$, $C = 23.33\ kN/m^2$

$D = 3.33\ kN/m^2$

(b) Additional load at the centre of the pier, so that there is no tension anywhere in the pier section.

Let W = Additional load (in kN) placed at the centre for no tension anywhere in the pier section.

The above load is compressive and will cause a compressive stress $= \frac{W}{A} = \frac{W}{12}\ kN/m^2$

As this load is placed at the centre, it will produce a uniform compressive stress across the section of the pier. But we know that there is tensile stress at point A having magnitude $= 10\ kN/m^2$. Hence the compressive stress due to load W should be equal to tensile stress at A.

$$\therefore \frac{W}{12} = 10$$

$$\therefore W = 10 \times 12 = 120\ kN$$

(c) Stresses at the corners with the additional load at the centre

Stress due to additional load $= \frac{W}{A} = \frac{120}{12} = 10\ kN/m^2$

(Compression)

This stress is uniform across the c/s of the pier. Hence to find the stresses at the corner with this additional load, we must add the stress $10\ kN/m^2$ in each value of the stresses already existing in the corners.

$\therefore$ stress at A, $\sigma_A = -10 + 10 = 0.$

Similarly $\sigma_b = 10 + 10 = 20\,kN/m^2$

$\sigma_c = 23.33 + 10 = 33.33\,kN/m^2$

$\sigma_D = 3.33 + 10 = 13.33\,kN/m^2$

Middle Third Rule for Rectangular section (ie Kernel of section).:-

The cement concrete column are weak in tension hence the load must be applied on this column in such a way that there is no tensile stress anywhere in the section but when an eccentric load is acting on a column it produces direct stress as well as bending stress. the result stress at any point in the section is the algebraic sum of the direct stress & bending stress

Consider a rectangular section of width 'b' and depth 'd' as shown in fig.

Let this section is subjected to a load which is eccentric to the axis Y-Y.

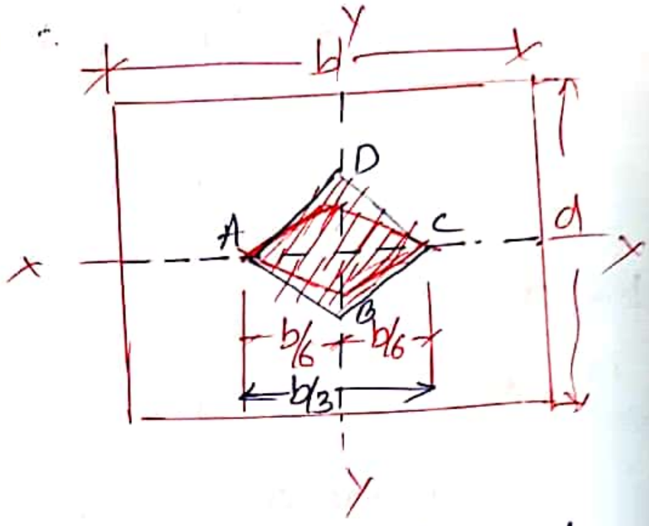
Let P = Eccentric load acting on the column

e = Eccentricity of the load

A = Area of the section

Then ~~from~~ equation we have minimum stress

$$\boxed{\sigma_{min} = P/A \left(1 - \frac{6e}{b}\right)} \longrightarrow (1)$$



If σ_{min} is -ve, then stress will be tensile. But if σ_{min} is zero (or positive) then there will be no tensile stress

Dam and Retaining walls

A large quantity of water is required for Irrigation and power generation throughout the year, A dam is constructed to store the water; A Retaining wall is constructed to retain the earth in hilly areas. The water stored in a dam, exerts pressure force on the face of the dam in contact with water. ''ly the earth, retained by a retaining wall, exerts pressure on the retaining wall, In ~~this chapter~~.

Types of Dam :-

. There are many types of dams but the following types of dams are more Important.

—

① Rectangular
② Trapezoidal dam
ⓐ water face vertical
ⓑ water face inclined

A trapezoidal dam as compare to rectangular dam is economical & easier to construct. Hence these days trapezoidal dams are mostly constructed.

Rectangular dam :-

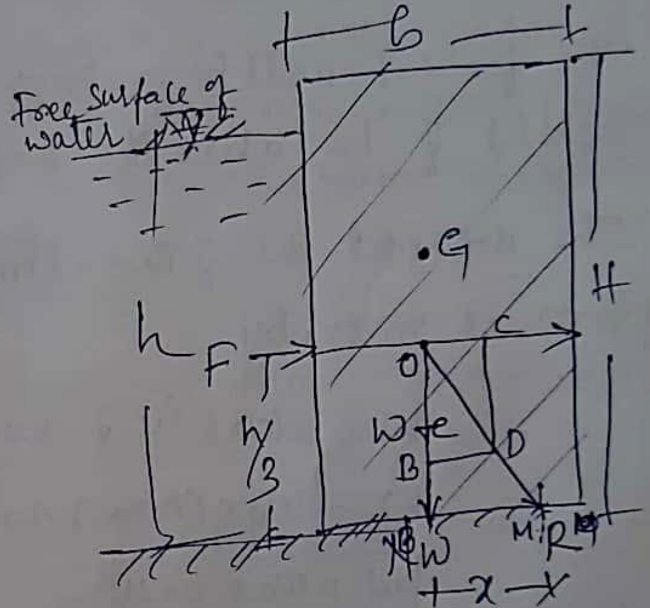


Fig shows rectangular dam having water on one of its sides

h = height of water
F = Force exerted by the water on the side of the dam
W = weight of dam/meter length of dam
H = height of dam
b = width of dam
w_0 = weight density of dam
consider 1m length of the dam
The force acting on the dam are

(i) The force F due to water in contact with the side of the dam

The Force F is given by $\boxed{F = wA\bar{h}}$

$$= w \times (h \times 1) \times \frac{h}{2} \quad (\because A = h \times 1 \text{ \& } \bar{h} = h/2)$$

$$\boxed{F = \frac{wh^2}{2}}$$

The force F will be acting horizontally at the height of h/3 above the base as shown in fig.

(ii) The weight W of the dam. The weight of the dam is given by

W = density of dam x volume of ~~weight~~ dam

W = w_0 x (Area of dam) x 1 ($\because$ length of dam = 1m)

$$\boxed{W = w_0 \times b \times H}$$

The weight W will be acting downwards through the C.G of the dam as show in fig.

There are only two forces are acting on the dam, the resultant force may be determine by the method of parallelogram of forces as shown in fig.

The force F is produced to intersect the line of action of the W at O Take OC = F

OB = W

To same scale. Complete the rectangle OBDC Then the diagonal OD will represent the resultant R to the same scale.

$\therefore$ Resultant $R = \sqrt{F^2+W^2}$.

and the angle made by the resultant with vertical is given by $\tan\theta = \frac{BD}{OB} = \frac{F}{W}$

Horizontal distance b/w the line of action of W & and the point through which the resultant cuts the base. In the fig diagonal OD represents of the F & W. Let the diagonal OD is extended so that its cuts the base of dam at point M. Also extend the line OB. so that it cuts the base at point N. Then the distance MN is the horizontal distance b/w the line of action of W & the point through the resultant cuts the base.

let x = Distance MN

The distance 'x' obtained from in's triangle OBD & ONM as given below

ie $\frac{NM}{ON} = \frac{BD}{OB}$ (or) $\frac{x}{h/3} = \frac{F}{W}$

$$\boxed{\therefore x = \frac{F}{W} \times \frac{h}{3}}$$

($\because$ Distance ON $= h/3$).
BD = OC = F
& OB = W

The distance 'x' can also be calculated by taking moments of all forces (here the Force F & W) about the point M. $\therefore$ $F \times \frac{h}{3} = W \times x$

$$\boxed{x = \frac{F}{W} \times \frac{h}{3}}$$

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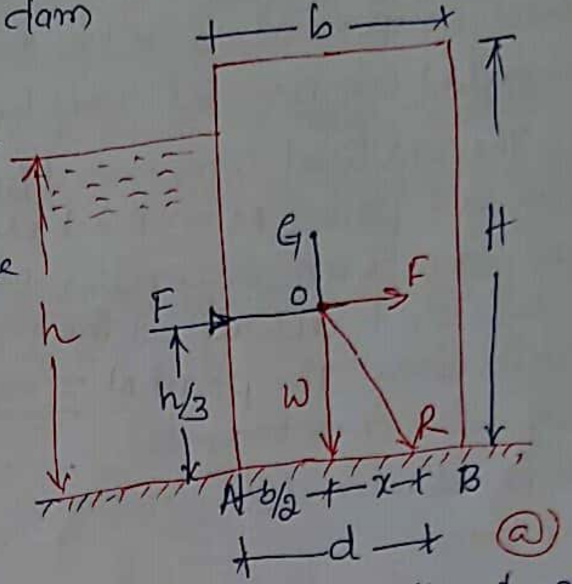
Stresses Across The Section of a Rectangular DAM

Fig shows a rectangular dam of height H and width b.

The dam is having water upto a depth of h.

The force acting on dam are

(i) The force F due to water at a height of h/3 above the base of the dam.



(ii) The weight W of the dam at the C.G of the dam.

The resultant force R is cutting the base of the dam at the point 'M' as shown in fig.

Let x = The horizontal distance b/w the line of action of W and the point Through which the resultant (R) cuts the base (i.e distance MN).

This distance is given by the equation.

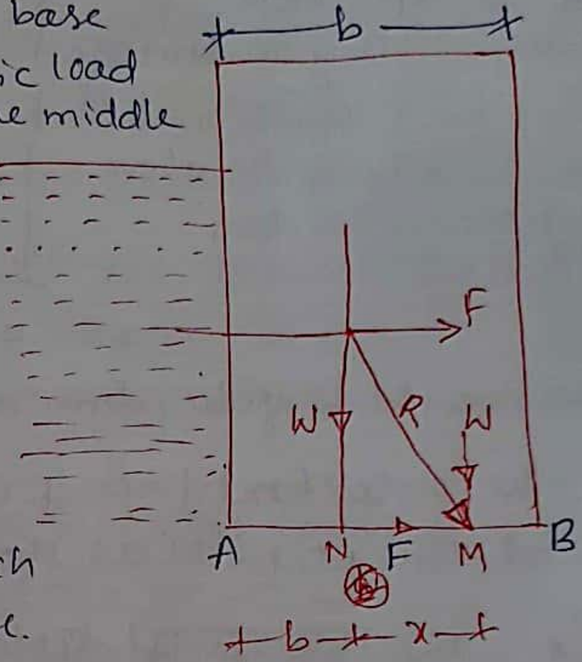
$$= \frac{F}{W} \times \frac{h}{3}$$

d = The distance b/w A & the point M, where the resultant 'R' cuts the base.

= Distance AM = AN + NM

$$\boxed{d = b/2 + F/W \times \frac{h}{3}}$$ ($\because$ Distance AN = Half the width of dam).

The resultant R meets the base of the dam at point M. This resultant force R acting at M may be resolved into vertical & horizontal components. The vertical component will be equal to W whereas the horizontal component will be equal to F as shown in fig. The vertical component W acting at point M on the base of the dam is an eccentric load as it is not acting at the middle of the base. The point N in fig. for a rectangular dam is the middle point of the base.



(b)

∴ Eccentricity of the vertical component W is equal to distance NM which is equal to x in this case.

∴ ~~Eccentricity~~, there will be moment,

Eccentricity e = Distance x → (a)

$= AM - AN = d - \frac{b}{2}$ → (b)

Due to the eccentricity, there will be a moment on the base of the dam. This moment will ~~be~~ cause some bending stresses at the base section of the dam.

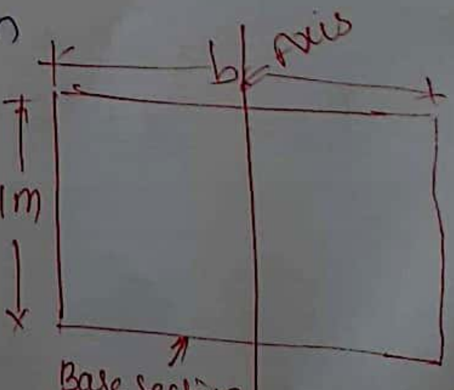
Now the moment on the base section

$= W \times$ Eccentricity

$= W \times e$

∴ moment ; $M = W . e$

We know that $\frac{M}{I} = \frac{\sigma_b}{y}$ → (1)



(c)

where M = moment

I = moment of Inertia

$$I = \frac{1 \times b^3}{12} = \frac{b^3}{12}$$

σ_b = Bending stress at a distance 'y' from the centre of gravity of the base section

y = Distance b/w the C.G of the base section & extreme edge of the base (which is equal to $\pm b/2$ in this case).

Substitute the values in equation(i). we get

$$\frac{W.e}{(b^3/12)} = \frac{\sigma_b}{(\pm b/2)}$$

$$\therefore \boxed{\sigma_b = \pm W.e \frac{b}{2} \times \frac{12}{b^3} = \pm \frac{6W.e}{b^2}}$$

The bending stress across base at point B.

$$= \frac{6We}{b^2}$$

And the bending stress across base at point A.

$$= -\frac{6W.e}{b^2}$$

But the direct stress on the base section due to direct load is given by

$$\sigma_d = \frac{\text{weight of dam}}{\text{Area of base}} = \frac{W}{b \times 1} = \frac{W}{b}$$

$\therefore$ Total stress across the base at B

$$\sigma_{max} = \sigma_d + \sigma_d = \frac{W}{b} + \frac{6W.e}{b^2} = \frac{W}{b}\left(1 + \frac{6.e}{b}\right) \rightarrow ⓒ$$

and total stress across the base at A.

$$\sigma_{min} = \sigma_d - \text{Bending stress at A}$$
$$= W/b - \frac{6W.e}{b^2} = \frac{W}{b}\left(1 - \frac{6.e}{b}\right) \rightarrow ⓓ$$

If the value of σ_{min} is -ve, this means that at the point A the stress is tensile.

① A masonry dam of rectangular section, 22m height & 11m width, has water upto height of 18m on its one side

Find:

(1) Pressure due to water F

② eccentricity at a distance x

③ Find minimum & maximum stress.

Take weight of density of masonry, 20 kN/m³.

Soln

$H = 22m$

$h = 18m$

$b = 11m$

$w_0 = 20 kN/m^3$

$w = 9.81 kN/m^3$

(i) Pressure force due to water on one meter length of dam.

let $F = w A \bar{h}$

$= 9.81 \times 1000 \times (18 \times 1) \times \frac{18}{2}$

$\boxed{F = 15.89 \times 10^5 N}$

(ii) The distance 'x' @ e

$x = \frac{F}{W} \times \frac{h}{3}$

$\therefore W = w_0 \times b \times H \times 1$

$= 20 \times 11 \times 22$

$W = 48.40 \times 10^5 N$

$x = \frac{15.89 \times 10^5}{48.40 \times 10^5} \times \frac{18}{2}$

$\boxed{x = 2.95 m}$

maximum stress at the base of the dam (i.e. σ_{max})

$$\sigma_{max} = \frac{W}{b}\left(1+\frac{6.e}{b}\right) = \frac{48.40\times10^5}{11}\left(1+\frac{6\times2.95}{11}\right)$$

$$= 4.4\times10^5(1+1.609)$$

$$\sigma_{max} = 11.48\times10^5\ N/m^2$$

$$= 1.148\ N/mm^2 \text{ (compressive)}$$

minimum stress at the base of the dam :-

$$\sigma_{min} = \frac{W}{b}\left(1-\frac{6.e}{b}\right) = \frac{48.40\times10^5}{11}\left(1-\frac{6\times2.95}{11}\right)$$

$$= 4.4\times10^5(1-1.609)$$

$$\sigma_{min} = -2.68\times10^5\ N/m^2 \ \text{(or)}\ 0.26\ N/mm^2$$

Tensile

(2) A masonry dam of rect section. 13m height & 6.5m width has water upto the 10m. If the weight density of masonry 20 kN/m³.

(i) Find Pressure due to F & W

(ii) Eccentricity at a distance 'x'

(iii) Find minimum & maximum stress.

Sol. $H = 13m$, $h = 10m$, $b = 6.5m$.

$W_0 = 20\ kN/m^3$.

$w = 9.81\ kN/m^3$.

$$F = 9.81\times1000\times10\times\frac{10}{2} = 49.05\times10^4\ N$$

$$W = 20\times1000\times6.5\times13 = 16.9\times10^5\ N$$

$$x = \frac{49.05\times10^4}{16.9\times10^5}\times\frac{10}{3} = 0.290\times\frac{10}{3} = 0.967\ m$$

$$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{16.9\times10^5}{6.5}\left(1 + \frac{6\times0.967}{6.5}\right)$$

$$\sigma_{max} = 2.725\,(1 + 0.\cancel{936}\,892)$$

$$\sigma_{max} = 5.15\times10^5\ N/m^2 \ @\ \underline{0.515\ N/mm^2}$$

$$\sigma_{min} = \frac{16.9\times10^5}{6.5}\left(1 - \frac{6\times0.967}{6.5}\right)$$

$$\sigma_{min} = 2.725\,(1 - 0.892)$$

$$\sigma_{min} = 0.294\times10^5\ N/m^2 \ @\ \underline{0.\ 0.029\ N/mm^2}$$

* Trapezoidal Dam having Water Face vertical

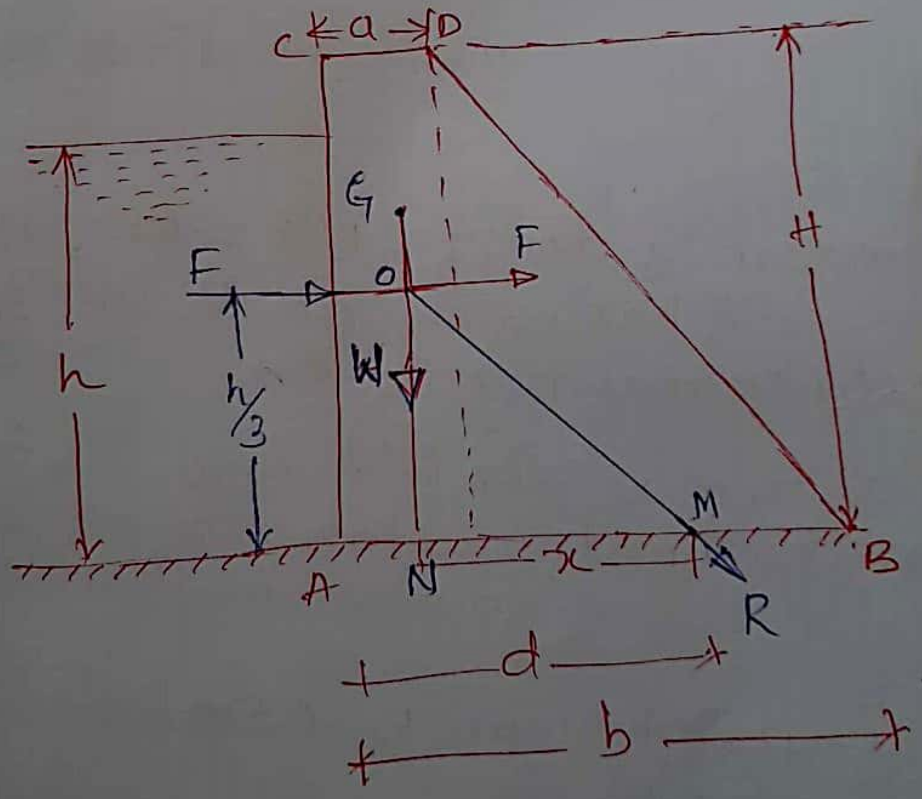


Fig shows a trapezoidal dam having water face vertical. Consider one metre length of the dam.

Let H = Height of dam
h = Height of water
a = Top width of dam,
b = Bottom width of dam,
w_0 = weight density of dam masonry,
w = weight density water $= \rho \times g = 1000 \times 9.81\ N/m^3$
$= 9.81\ kN/m^3 = 9810\ N/m^3$
F = Force exerted by water
W = Weight of dam / metre length of dam

(i) F = Force exerted by water
$= w \times A \times \bar{h} = w \times (h \times 1) \times \frac{h}{2} = \frac{wh^2}{2}$

The force F will be acting horizontally at a height of $h/3$ above the base.

(ii) W = weight of dam per metre length of dam
= weight density of dam × (Area of C/S) × 1
$= w_0 \times \left(\frac{a+b}{2}\right) \times H \times 1$ ($\because$ Area $= \frac{1}{2}$ (Sum of parallel sides) × Height.

$$\boxed{W = w_0 \times \left(\frac{a+b}{2}\right) \times H}$$

(iii) The weight W will be acting downwards through the C.G of the dam.

(i) The distance of the C.G of the trapezoidal section from the vertical face AC is obtained by splitting the dam section into a rectangle and a triangle, taking the moments of their areas about line AC and equating the same with the moment of the total area of the trapezoidal section about the line AC.

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ie Area of rectangle × Distance of C.G of rectangle from AC + Area of triangle × Distance of C.G of triangle from AC = Total area of trapezoidal × Distance AN

or $(a \times H) \times \frac{a}{2} + \frac{(b-a) \times H}{2}\left(a + \frac{b-a}{3}\right) = \left(\frac{a+b}{2}\right) \times H \times AN$

from the above equation distance AN can be calculated.

(ii) The distance AN can also be calculated by using the relation given below.

$$AN = \frac{a^2 + ab + b^2}{3(a+b)} \longrightarrow (1)$$

Now let x = Horizontal distance between the line of action weight of dam and the point where the resultant cuts the base.

$$\boxed{x = \frac{F}{W} \times \frac{h}{3}}$$

d = Distance b/w A & the point M where the resultant cuts the base (i.e, distance AM)

$$d = AN + NM \longrightarrow (2)$$

The distance AN + NM can be calculated and hence the distance d will be known.

Now the eccentricity, $e = d -$ half the base width of dam

$$= d - \frac{b}{2}$$

Then the total stress across the base of the dam at point B,

$$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right) \longrightarrow (3)$$

and the total stress across the base at A.

$$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right) \longrightarrow (4).$$

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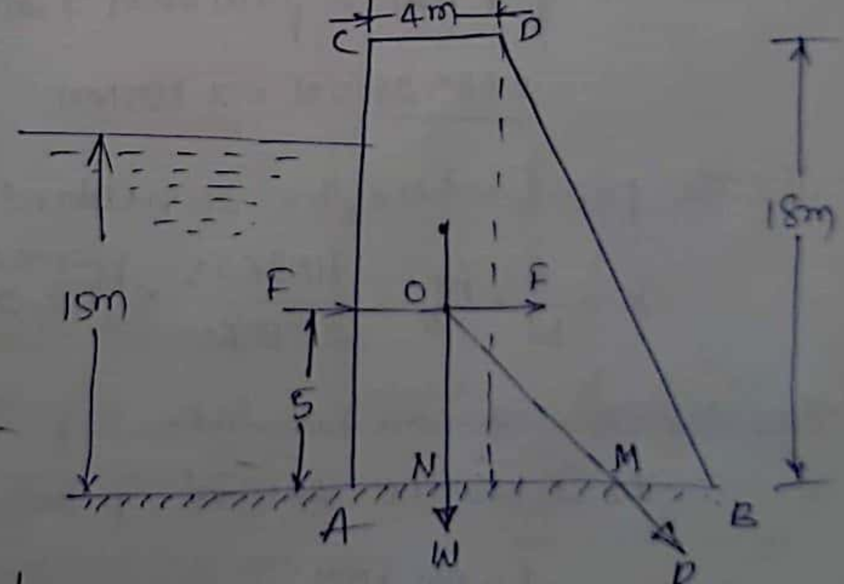
① A trapezoidal masonry dam is of 18m height. The dam is having water upto a depth of 15m on its vertical side. The top + bottom width of the dam are 4m + 8m respectively. The weight density of the masonry is given as 19.62 kN/m3. Determine:

(i) The resultant force on the dam p/m length.
(ii) The point where the resultant cuts the base, +
(iii) The maximum + minimum stress intensities at the base.

Soln:-

Height of dam, $H = 18m$
Depth of water, $h = 15m$
Top width of water, $a = 4m$
Bottom width of dam, $b = 8m$
weight density of masonry.

$w_0 = 19.62\ kN/m^3 = 19620\ N/m^3$.



(i) Resultant force on dam
Let us find first the force F and weight of the dam.

Force $F = w \times A \times \bar{h}$
$= 9810 \times (h \times 1) \times \frac{h}{2}$
$= 9810 \times 15 \times 15/2 = 1103625\ N$

And it is acting at a distance of $h/3$ i.e. $15/3 = 5m$.

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Now weight of dam is given by

$$W = \text{weight density of masonry} \times \text{Area of dam} \times 1$$

$$= w_0 \times \left(\frac{a+b}{2}\right) \times H \times 1$$

$$= 19620 \times \left(\frac{4+8}{2}\right) \times 18 \times 1 = 2118960 N$$

[The distance of line of action of W from the line AC is obtained by splitting the dam into rectangle and triangle, taking the moment of their area about line AC & equating to the moment of the area of the trapezoidal about the line AC.]

(a) $$4 \times 18 \times 2 + \frac{4 \times 18}{2} \times \left(4 + \frac{1}{3} \times 4\right) = \left(\frac{4+8}{2}\right) \times 18 \times AN$$

$$144 + 36\,[5.33] = 108 \times AN$$

$$\therefore AN = \frac{144 + 36 \times 5.33}{108} = 3.11 m.$$

$\therefore$ The Resultant force 'R' is given by

$$R = \sqrt{F^2 + W^2} = \sqrt{1103625^2 + 2118960^2}$$

$$= 2389225.5 N = 2.389 MN.$$

(ii) The point where the resultant cuts the base.

$$x = \frac{F}{W} \times h/3 = \frac{1103625}{2118960} \times \frac{15}{3} = 2.604 m$$

~~The distance x can be take~~

The distance $AM = d$

$$d = AN + NM$$

$$\boxed{d = 3.11 + 2.604 = 5.714 m}$$

Now eccentricity, $e = d - b/2$

$$= 5.714 - 8/2 = 1.714 m$$

(ii) The maximum & minimum stress intensities.

Let σ_{max} = Maximum stress &

σ_{min} = Minimum stress.

$$\sigma_{max} = \frac{W}{b}\left(1+\frac{6e}{b}\right) = \frac{2118960}{8}\left(1+\frac{6\times1.714}{8}\right)$$

$$= 264870\,(1+1.2855) = 605360\ N/m^2$$

$$\sigma_{min} = \frac{W}{b}\left(1-\frac{6\times e}{b}\right) = \frac{2118960}{8}\left[1-\frac{6\times1.714}{8}\right]$$

$$= 264870\,(1-1.2855) = -\underline{75620}\ N/m^2$$

(2) A masonry trapezoidal dam 4m high, 1m wide at its top and 3m width its bottom retains on its vertical face. Determine the maximum and minimum stresses the base.

(i) when the reservoir is full &

(ii) when the reservoir is empty. Take the weight density of masonry as 19.62 kN/m³.

Soln:

$H = 4m$

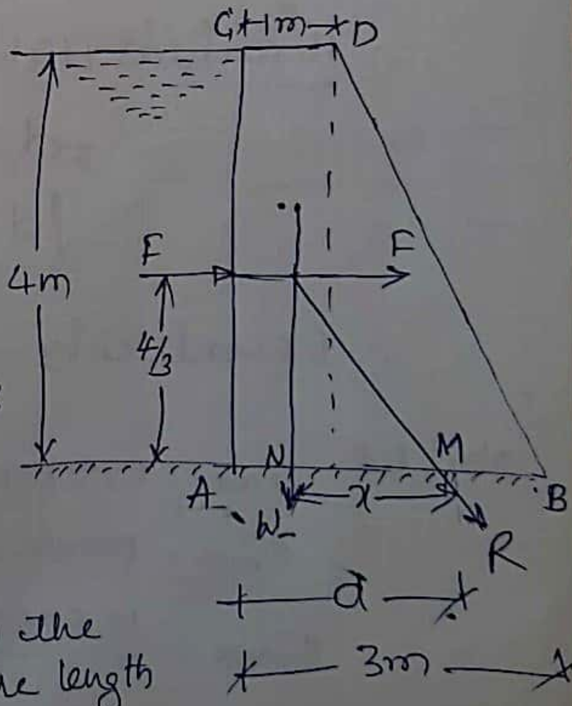
Top width $a = 1m$

Bottom $b = 3m$

Depth of water, $h = 4m$

weight density of masonry,

$w_0 = 19.62\ kN/m^3 = 19620\ N/m^3$



Consider one metre length of dam.

(i) where reservoir is full of water

The force exerted by water on the vertical face of the dam/metre length

$$F = w\times A\times\bar{h} = 9810\times(4\times1)\times\frac{4}{2} = \underline{78480\,N}$$

$(\because w = 9810\ N/m^3$ for water$)$

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The weight of dam per metre length is given by.

W = weight density of masonry dam × Area of trapezoidal × 1

$= w_0 \times \left(\frac{a \times b}{2}\right) \times H$

$= 19620 \times \left(\frac{1+3}{2}\right) \times 4 = \underline{\underline{156960 N}}.$

Now let us find the position of the C.G

$\therefore \left(4 \times 1 \times \frac{1}{2}\right) + \left[\frac{4+2}{2} \times \left(1 + \frac{1}{3} \times 2\right)\right] = \left(\frac{1+3}{2}\right) \times 4 \times AN$

$2 + 4 \times 1.67 = 8 \times AN$

$\boxed{AN = 1.08 m}$

AN can also be calculated calculated.

$AN = \frac{a^2 + ab + b^2}{3(a+b)} = \frac{1^2 \times 1 \times 3 + 3^2}{12} = \frac{13}{12} = \underline{\underline{1.08 m}}$

Find, $x = \frac{F}{W} \times \frac{h}{3} = \frac{78480}{156960} \times \frac{4}{3} = 0.67 m.$

$\therefore$ Horizontal distance AM

$d = AN + x$

$\boxed{d = 1.08 + 0.67 = 1.75 m}$

$\therefore$ Eccentricity $e = d - \frac{b}{2} = 1.75 - 1.50 = \underline{\underline{1.25 m}}$

Now let σ_{max} = maximum stress at the base of the dam,

σ_{min} = minimum stress.

$\sigma_{max} = \frac{156960}{3}\left(1 + \frac{6 \times 0.25}{3}\right) = 78480 N/m^2.$

$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right) = \frac{156960}{3}\left(1 - \frac{6 \times 0.25}{3}\right)$

$= \underline{\underline{26163}} N/m^2$

When water is empty :-

F is zero

$W = 156960 N$

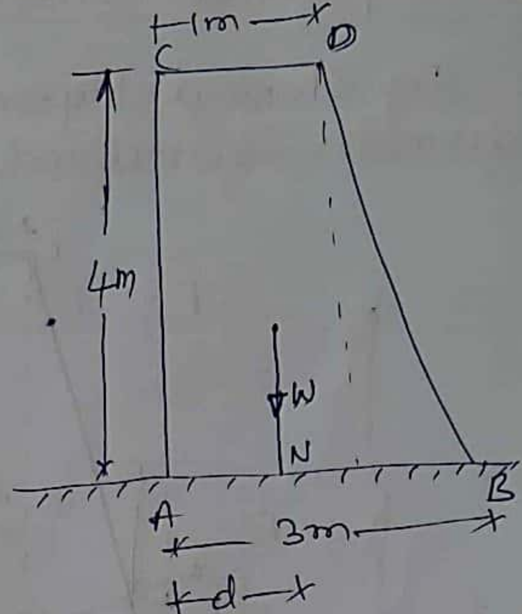
$\therefore$ Distance AN $= 1.08m$

$d = 1.08 m$

Now. $e = d - \frac{b}{2}$

$= 1.08 - 3/2$

$\boxed{e = -0.42m}$



$\therefore \sigma_{max} = \frac{W}{b}\left(1 + \frac{6 \times e}{b}\right)$

(-ve sign only indicates that stress at A will be more than at B).

(e = 0.42 only)

$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6 \cdot e}{b}\right)$

$= \frac{156960}{3}\left(1 + \frac{6 \times 0.42}{3}\right) = 96265 \ N/m^2$

$\sigma_{min} = W/b\left(1 - \frac{6 \cdot e}{b}\right) = \frac{156960}{3}\left(1 - \frac{6 \times 0.42}{3}\right) = 8367.63 \ N/m^2$

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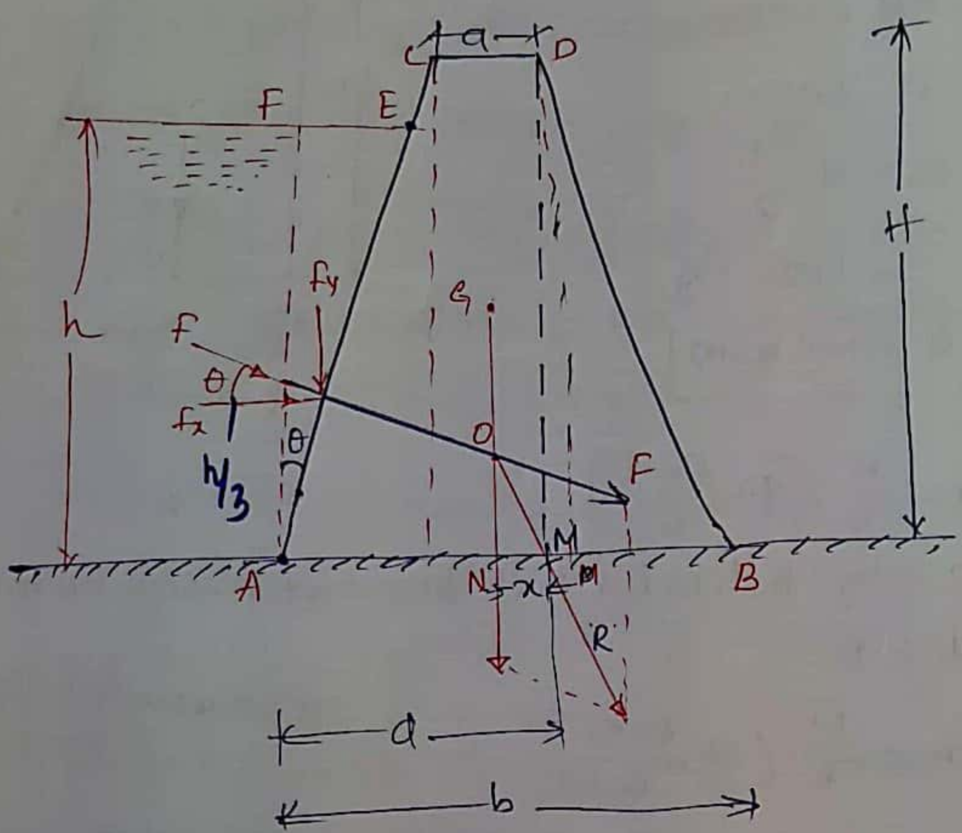
Trapezoidal Dam having water face inclined

Fig shows a trapezoidal dam section having its water face inclined.



let H = Height of dam,

h = Height of water.

a = Top width of dam.

b = bottom width of dam,

w_0 = weight density of dam masonry.

w = weight density of water $= 9810\ N/m^3$

θ = Inclined of face AC, with vertical.

F_x = component of F in x-direction

$= F\cos\theta$

F_y = component of F in vertically downward direction

$= F\sin\theta$.

W = weight of dam per metre length of dam,

$$= \frac{w \times h^2}{2} \times \frac{EF}{AF}$$

$$= \frac{w \times h^2}{2} \times \frac{EF}{h}$$

$$= w \times \frac{h \times EF}{2} \qquad (\because AF = h).$$

$$= w \times \text{Area of } \Delta^{le} AEF \qquad \left(\because \text{Area of } \Delta^{le} AEF = \frac{EF \times h}{2}\right)$$

$$= w \times \text{Area of triangle } AEF \times 1$$

$$\boxed{F_y = \text{weight of water in the wedge } AEF}$$

Hence the force F acting on inclined face AE is equalent to force F_x acting on the vertical face AF and force F_y which is equal to the weight of water in the wedge AEF.

The force F_x acts at a height $h/3$ above the base whereas the force F_y acts through the C.G. of the triangle AEF.

(ii) weight of dam/metre length of the dam and it is given by

$$W = \left(\frac{a+b}{2}\right) \times H \times w_0$$

The weight W ~~at~~ will be acting through the C.G. of the trapezoidal section of the dam. The distance of the C.G. of the trapezoidal section shown in fig from the point A is obtained by splitting the dam section into triangles and rectangle. taking the moments of their areas about the point A. By doing so the distance AN will be known.

(iii) The force R, which is the resultant of the forces F & W, cuts the base of the dam at point M. The distance AM can be calculated by taking moments of all forces (ie, forces F_x, F_y & W) about the point M. But the distance AM = d

Now the eccentricity $e = d - b/2$

Then the total stress across the base of the dam at point B,

$$\sigma_{max} = \frac{V}{b}\left(1 + \frac{6 \cdot e}{b}\right)$$

and the total stress across the base of the dam at point A.

$$\sigma_{min} = \frac{V}{b}\left(1 - \frac{6 \cdot e}{b}\right)$$

where V = Sum of the vertical forces acting on the dam.

$= F_y + W$

① A masonry dam of trapezoidal section is 10m height. It has top width of 1m and bottom width 7m. The face exposed to water has a slope of 1 horizontal to 10 vertical. Calculate the maximum & minimum stresses on the base when the water level coincides with the top of the dam. Take weight density of masonry as 19.62 kN/m³.

Given:-

Height of dam, $H = 10$ m

Top width $a = 1$ m

Bottom width of dam $b = 7$ m

Slope of face exposed to water = 1 hor to 10 vertical

∴ length of EC = 1m

Depth of water = 10m

weight density of masonry, $w_0 = 19.62$ kN/m² $= 19620$ N/m³

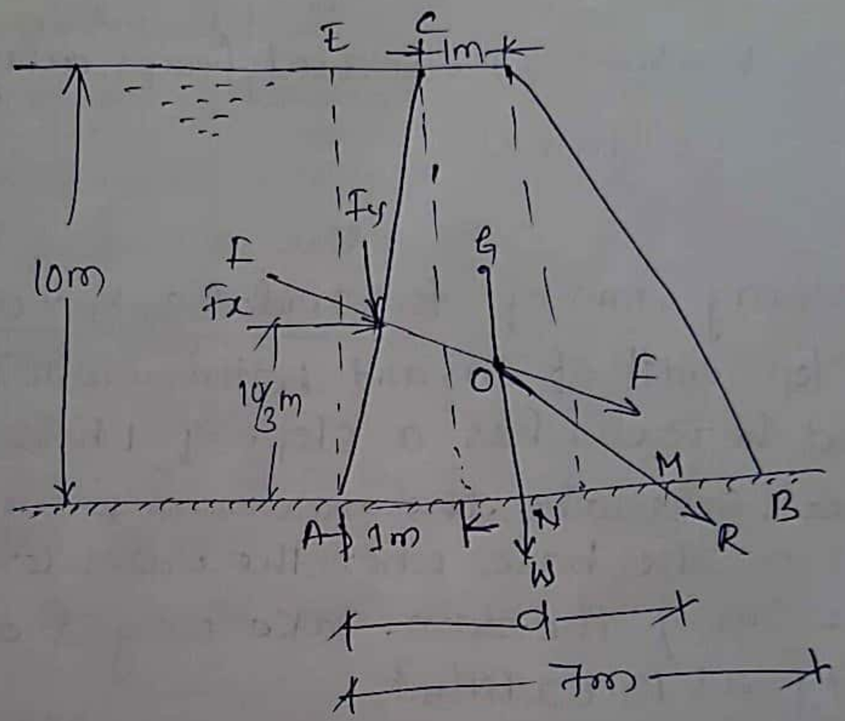
consider the one metre length of the dam.

Let the weight of the dam (W) cut the base at 'N' where the Resultant 'R' cuts the base at M.

The force F due to water acting on the face AC is resolved into two components F_x & F_y as shown in fig.

But F_x = Force due to water on vertical face AE

$= w \times A \times \bar{h}$



$= 9810 \times (10 \times 1) \times \frac{10}{2}$ $\quad$ ($\because$ Area A = AE $\times$ 1)

$\boxed{F_x = 490500N}$

The force F_x will act at a height of $\frac{10}{3}$ m above the base of the dam.

F_y = weight of water in wedge AEC.

$= w \times$ Area of AEC $\times 1$

$= 9810 \times \frac{10 \times 1}{2} \times 1 = \underline{49050N}$

The force F_y will act downward through the C.G of the Δ^{le} AEC i.e. at a distance $\frac{1}{3} \times 1 = \frac{1}{3}$ m from AO.

weight of dam $W = w_0 \times \left(\frac{a+b}{2}\right) \times H = 19620 \times \left(\frac{1+7}{2}\right) \times 10$

$= \underline{\underline{784800N}}$

The weight W will be acting through the C.G. of the dam.

The position of C.G of the dam (i.e distance AN) is obtained by splitting the trapezoidal into triangles & $\square^{les}$, taking the moments of their area about A & equating to the moment of area of the trapezoidal about the point A.

$\therefore \left(\frac{10 \times 1}{2} \times \frac{2}{3}\right) + (10 \times 1 \times 1.5) + \frac{10 \times 5}{2} \times (2 + 5/3) = \left(\frac{a+b}{2}\right) \times H \times AN$

(or) $3.33 + 15 + 91.67 = \left(\frac{1+7}{2}\right) \times 10 \times AN = 40 \times AN$

$\therefore \boxed{AN = \frac{110}{40} = 2.75m}$

The resultant force R cuts the base at M. To find the distance of M from A (i.e. distance AM). take the moments of all forces out the point M.

$\therefore F_x \times \frac{10}{3} - F_y \times (AM - 0.33) - W \times NM = 0$

$490500 \times 10/3 - 49050 \times (AM - 0.33) - 784800 \times (AM - AN) = 0$

$\therefore (NM = AM - AN)$

$\frac{490500}{3} - 490500 AM + 16350 - 784800 AM + 784800 \times 2.75 = 0.$

$(\because AN = 2.75m)$

(a) $\frac{490500}{3} + 16350 + 784800 \times 2.75 = 784800AM + 49050AM$

$3809550 = 833850AM$

$AM = \frac{3809550}{833850} = 4.568$

$$\boxed{d = 4.568m}$$

Now the eccentricity, $e = d - b/2$

$e = 4.568 - 7/2 = 1.068m$

Maximum & minimum stresses on the base.

Let σ_{max} = maximum stress on base.

$\sigma_{max} = V/b \left(1 + \frac{6.e}{b}\right)$

V = Total vertical forces on the dam.

$= W + F_y = 78480 + 49050 = 833850N$

$\therefore \sigma_{max} = \frac{833850}{7}\left(1 + \frac{6 \times 1.068}{7}\right)$

$= 228167 N/m^2$

$$\boxed{\sigma_{min} = \frac{833850}{7}\left(1 - \frac{6 \times 1.068}{6}\right)} = 10077.8 N/m^2$$

(2) A masonry dam of trapezoidal section is 10m high. It has top width of 1m & bottom width 6m. The face exposed to water has slope of 1 horizontal to 10 vertical. Calculate the maximum & minimum stresses on the base when water level coincides with the top of the dam. Take weight density of masonry as 22.563kN/m³.

Ans:-

Height of dam, $H = 10m$

Height of water, $h = 10m$

Top width of dam, $a = 1m$

Bottom width of dam, $b = 6m$

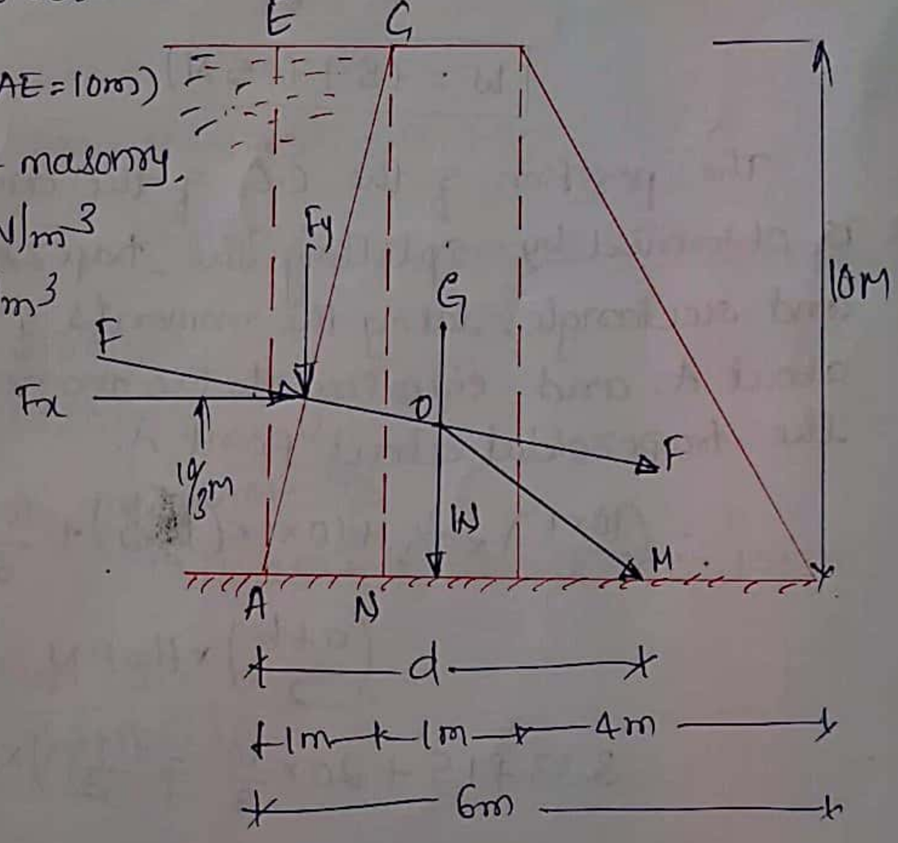
Slope of the face AC which is exposed to water = hor. to 10 vertical.

$\therefore EC = 1m \quad (\because AE = 10m)$

weight density of masonry,

$w_0 = 22.563\ kN/m^3$

$= 22563\ N/m^3$



consider one metre length of dam.

Now the force F due to water acting on the face AC is resolved into two components Fx & Fy as shown in fig.

Force, F_x = Force due to water acting on vertical face AE

$= w \times A \times \bar{h}$

$= 9810 \times (10 \times 1) \times \frac{10}{2} \quad (\because \bar{h} = \frac{10}{2})$

$\boxed{F_x = 490500N}$

and Force, F_y = weight of water in the wedge AEC

$$= w \times \text{Area of } \Delta^{le} AEC \times 1$$

$$= w \times \frac{EC \times AE}{2} \times 1$$

$$= 9810 \times \frac{1 \times 10}{2} \times 1 = 49050N$$

weight of dam,

$$W = w_0 \times \left(\frac{a+b}{2}\right) \times H = 22563 \times \left(\frac{1+6}{2}\right) \times 10$$

$$\boxed{W = 789705N}$$

The position of the C.G of the dam (i.e, distance AN) is obtained by splitting the trapezoidal into triangles and rectangle, taking the moments of their areas about A, and equating to the moment of the area of the trapezoidal about point A.

$$\therefore \left(\frac{10 \times 1}{2}\right) \times \frac{2}{3} + 10 \times 1 \times \left(1 + \frac{1}{2}\right) + \frac{10 \times 4}{2} \times \left(1 + 1 + \frac{1}{3} \times 4\right)$$

$$= \left(\frac{a+b}{2}\right) \times H \times AN$$

$$3.33 + 15 + 20 \times \frac{10}{3} = \left(\frac{1+6}{2}\right) \times 10 \times AN$$

$$85 = 35 \times AN$$

$$\therefore AN = 85/35 = 2.43m$$

Now let the resultant R of forces F & W cut the base at M.

Taking the moment of all forces (i.e. F_x, F_y & W) about the point M, we get

$$F_x \times 10/3 = W \times NM + F_y \times \left(AM - \frac{1}{3} \times 1\right)$$

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$$490500 \times \frac{10}{3} = 789705 \times (AM - AN) + 49050\left(AM - \frac{1}{3}\right)$$

$$\frac{4905000}{3} = 789705 \times Am - 789705 \times AN + 49050 \times Am - \frac{49050}{3}$$

$$= Am \times (789705 + 49050) - 789705 \times \frac{17}{7} - \frac{49050}{3} \quad \left(\because AN = 17/7\right)$$

$$= AM \times 838755 - 1917855 - \frac{49050}{3}$$

$$\therefore AM \times 838755 = \frac{4905000}{3} + 1917855 + \frac{49050}{3} = 3569205$$

$$\therefore Am = \frac{3569205}{838755} = 4.255m$$

$\therefore$ Eccentricity, $e = AM - b/2$

$$= 4.255 - 6/2 = 4.255 - 3.0 = \underline{1.255m}$$

<u>maximum stress on the base</u> :-

$$\sigma_{max} = V/b\left(1 + \frac{6.e}{b}\right)$$

where V = Total vertical forces on the dam

$$= W + F_y = 789705 + 49050 = \underline{838755N}$$

$$\therefore \sigma_{max} = \frac{838755}{6}\left(1 + \frac{6+1.255}{6}\right) = 315232\ N/m^2$$

$$\sigma_{min} = V/b\left(1 - \frac{6.e}{b}\right)$$

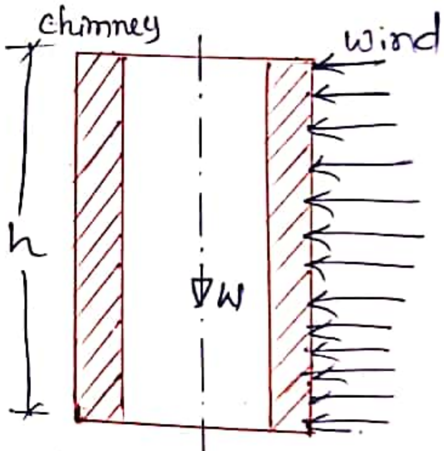
$$= \frac{838755}{6}\left(1 - \frac{6+1.255}{6}\right) = \underline{35647\ N/m^2}$$

Chimneys

Chimneys are tall structures subjected to horizontal wind pressure. The the base of the chimneys are subjected to horizontal wind pressure. The base of the chimneys are subjected to bending moment due to horizontal wind force. This bending moment at the base produces bending stresses. The base of the chimney is also subjected to direct stresses due to self-weight of the chimney. Hence at the base of the chimney the bending stress and direct stress are acting. The direct stress σ_0 is given by.

$$\sigma_d = \frac{\text{weight of chimney}}{\text{Area of s/c at the base}}$$

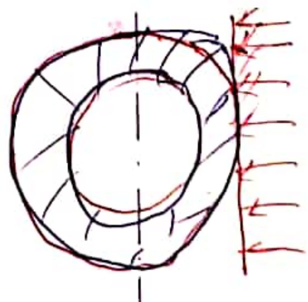
$$\boxed{\sigma_d = \frac{W}{A}}$$



The bending stress (σ_b) is obtained from

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M}{I} \times y = \frac{M}{\left(I/y\right)} = \frac{M}{Z}$$



Chimney subjected to wind force

where M = bending moment due to horizontal wind force and

Z = modulus of section.

The wind force (F) acting in the horizontal direction on the surface of chimney is given by.

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$$\boxed{F = K \times p \times A}$$

where K = co-efficient of wind resistance, which depends upon the shape of the area exposed to wind.

= 1 for rectangular & square chimneys

= 2/3 for circular chimney.

p = Intensity of wind pressure.

A = project area of the surface exposed to wind.

= D×h for circular chimney.

= b×h for rectangular @ square chimney

b = width of chimney exposed to wind.

h = height of chimney.

The wind force ~~acting~~ 'F' will be acting at $\frac{h}{2}$.

The moment of 'F' at the base of the chimney will be $F \times \frac{h}{2}$.

Hence bending moment (M) at the base of chimney is given by.

$$\boxed{M = F \times \frac{h}{2}}$$

① Determine the maximum & minimum stresses at the base of an hollow circular chimney of height 20m with external diameter 4m and internal diameter 2m. The chimney is subjected to a horizontal wind pressure of intensity 1kN/m2. The specific weight of the material of chimney is 22 kN/m3.

Soln:-

Height, $H = 20m$; External dia, $D = 4m$; Internal dia - $d = 2m$, Horizontal wind pressure, $p = 1 KN/m^2$

specific weight, $w = 22 KN/m^3$

Let us first find the weight of the chimney and horizontal wind force (F).

weight (W) of the chimney is given by.

$W = \rho \times g \times$ volume of chimney

$=$ weight density $\times$ volume of chimney

$= w \times$ [Area of C/S] $\times$ height

$$\boxed{W = 22 \times \pi/4 \, (4^2 - 2^2) \times 20 = 4146.9 \, KN}$$

$\therefore$ Direct stress at the base of the chimney.

$\sigma_d = \frac{W}{A}$ where $A =$ Area of C/s

$$= \frac{4146.9}{\pi/4\,(4^2 - 2^2)} = \frac{4146.9}{3\pi} = 440 \, KN/m^2$$

+ find the force (F).

$$F = K \times p \times A$$

where $K = 2/3$ as the section is circular.

$A =$ projected area of the surface exposed to wind.

$= D \times h$ where $D =$ External dia $= 4m$.

$= 4 \times 20 = 80 m^2$

$p =$ horizontal wind pressure $= 1 KN/m^2$

$$\therefore \quad F = 2/3 \times 1 \times 80 = \frac{160}{3} = 53.33 \, KN.$$

$H = 25m$, $D = 5m$, $d = 2.5m$, $P = 1 KN/m^2$, sp wt $= 23 KN/m^3$

The bending moment (M) at the base,

$$M = F \times h/2 = 53.33 \times 20/2 = \underline{533.3\ KN\text{-}m}$$

The bending stress (σ_b) is given by equation as

$$\sigma_b = M/Z \quad \text{where } Z = I/y$$

$$I = \frac{\pi}{64}(D^4 - d^4),\ y = \frac{D}{2}$$

$$I = \pi/64\,(4^4 - 2^4) = 11.78\,m^4 \ \&\ y = \frac{4}{2} = 2m.$$

$$Z = I/y = \frac{11.78}{2} = \underline{5.89\,m^3}$$

$$\boxed{\sigma_b = \frac{533.3}{5.89} = 90.54\ KN/m^2}$$

Now the maximum & minimum stresses at the base

$$\sigma_{max} = \sigma_d + \sigma_b = 440 + 90.54 = 530.54\ KN/m^2\ (Comp)$$

$$\sigma_{min} = \sigma_d - \sigma_b = 440 - 90.54 = \underline{349.46\ KN/m^2}\ (Comp).$$

Retaining wall

The wall which are used for retaining the soil or earth, known as retaining wall. The earth retained by retaining wall, exerts pressure on the retaining wall in the same way has water exerts pressure on the dam. A number of theories have been evolved to determine the pressure exerted by the soil @ earth on the retaining wall. one of the theory is Rankine's theory of earth pressure. before discussing ranking theory. let us define the angle of repose and study the equilibrium of a body on an inclined plane.

Angle of Repose :-

It is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane ~~who~~ by the assistance of friction only. The earth particles lack in cohesion and have definite angle of repose And angle of repose is equal to angle of friction (ϕ). Angle of friction is the angle made by the resultant of the normal reaction and limiting force of friction with the normal reaction. [friction:- the resistance that one surface @ object encounters when moving over another]

Equilibrium of a Body on an inclined body

If the inclination of the inclined plane is less than the angle or repose, the body will be in equilibrium entirely by friction only. But if the inclination of the plane is greater than the angle of repose, the body will be in equilibrium only with the assistance of an external force.

let an external horizontal force 'P' is applied on a body, which is placed on an inclined plane having inclination greater than angle of repose, to keep the body in equilibrium. There are two cases.

(i) The body may be on the point of moving down the plane. and

(ii) The body may be on the point of moving up the plane.

<u>1st case</u> :- The body is on the point of moving down the plane.

Let W = Weight of the body

P = Horizontal force applied on the body in order to prevent the body from moving down the plane.

θ = Angle of inclination of the plane

φ = Angle of limiting friction i.e., angle made by the resultant of normal reaction and limiting force of friction with the normal reaction as shown in fig. (b).

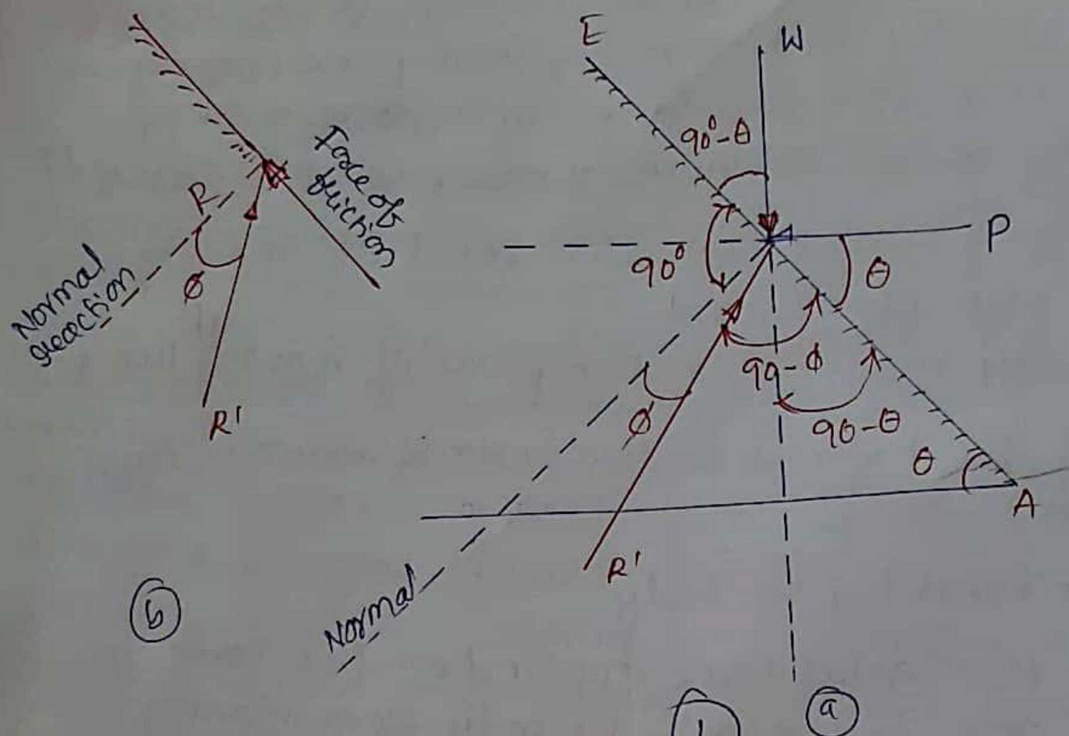
R' = Resultant of normal ~~action~~ reaction and limiting force of friction.

The force acting on the body are shown in fig (a). The body is in equilibrium under the action of three forces, W, P & R'. Applying lami's theorem to the forces acting on the body, we get

$$\frac{P}{\text{Sine of angle b/w } W \,\&\, R'} = \frac{W}{\text{Sine of angle b/w } R' \,\&\, P}$$

$$\frac{P}{\sin(90-\theta+90+\phi)} = \frac{W}{\sin(\theta+90-\phi)}$$

$$\frac{P}{\sin(180-(\theta-\phi))} = \frac{W}{\sin(90+\theta-\phi)}$$



Body moving down.

$$P = \frac{W \sin[180-(\theta-\phi)]}{\sin[90+(\theta-\phi)]} = \frac{W\sin(\theta-\phi)}{\cos(\theta-\phi)}$$

$$\boxed{P = W\tan(\theta-\phi)}$$

2nd case :- The body is on the point of moving up the plane.

Let W = Horizontal force applied on the body in order to prevent the body from moving up the plane.

θ = Angle of inclination.

ϕ = Angle of limiting friction i.e., angle made by the resultant (R') of normal reaction & limiting force of friction with the normal reaction as shown in fig (b).

R' = Resultant of normal reaction & limiting force of friction.

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The force acting on the body are shown in fig 2ⓐ.
The body is in equilibrium under the action of three forces W. P + R'

Applying lami's theorem to the forces acting on the body. we get

$$\frac{P}{\text{Sine of angle b/w } W + R} = \frac{W}{\text{Sine of angle b/w } R' + P}$$

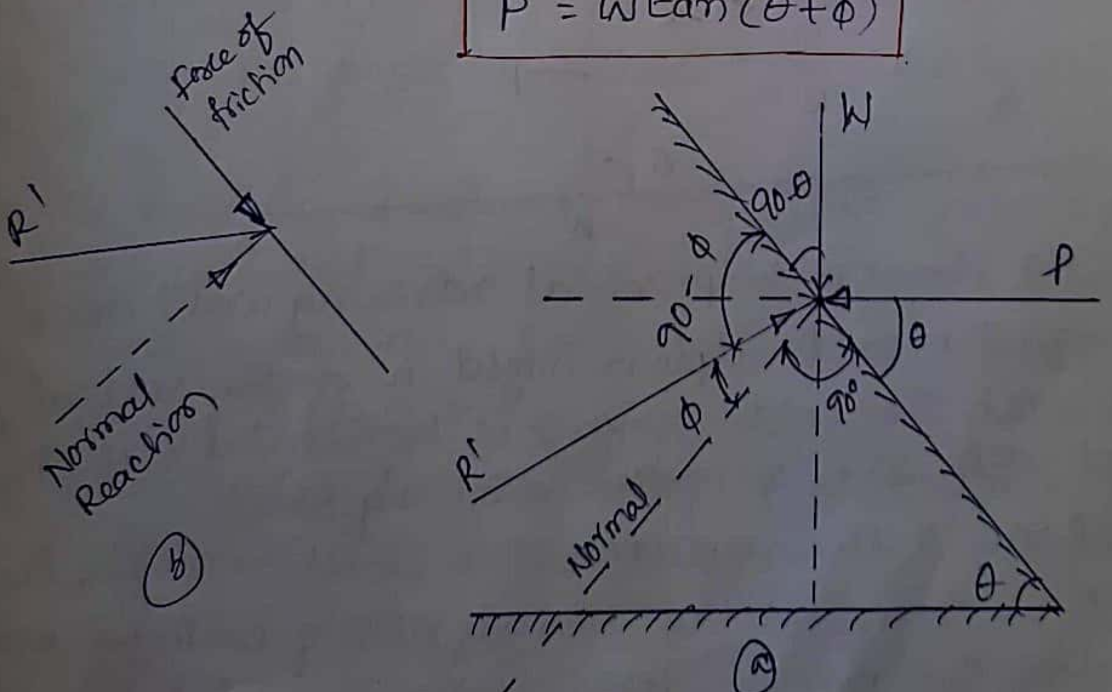
$$\frac{P}{\sin(90-\phi+90-\theta)} = \frac{W}{\sin(\theta+90+\phi)}$$

$$\frac{P}{\sin(180-(\theta+\phi))} = \frac{W}{\sin[90+(\theta+\phi)]}$$

$$P = \frac{W\sin[180-(\theta+\phi)]}{\sin[90+(\theta+\phi)]}$$

$$= \frac{W\sin(\theta+\phi)}{\cos(\theta+\phi)}$$

$$\boxed{P = W\tan(\theta+\phi)}$$



②

Body moving up

Rankine's theory of earth Pressure :-

Rankine's theory of earth Pressure is used to determine the Pressure exerted by the earth or soil on the retaining wall. This theory is based on the following Assumptions.

1. The earth or soil retained by a retaining wall is cohesionless.
2. Frictional resistance b/w the retaining wall and the retained material (i.e. earth or soil). is neglected.
3. The failure of the retained material takes place along a plane, Known as repture plane.

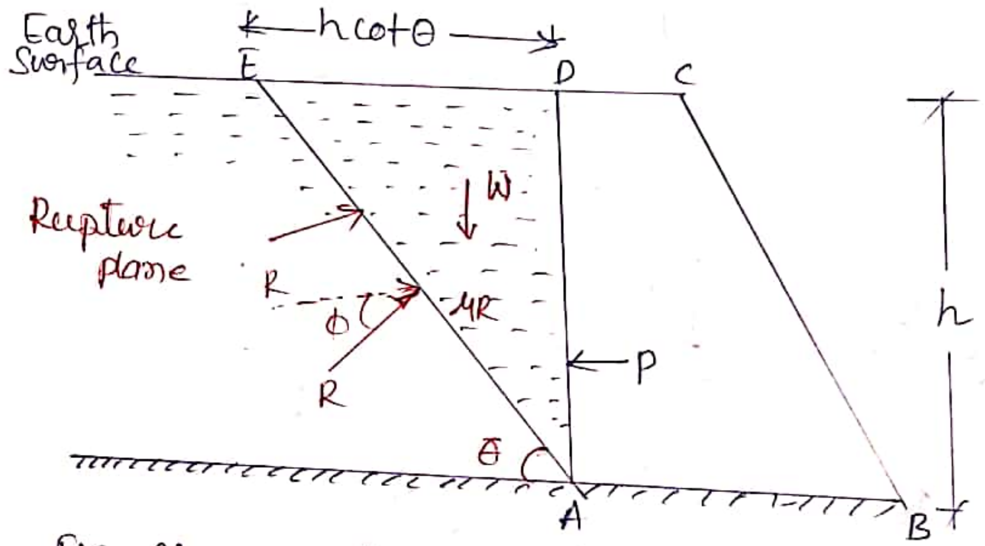


Fig shows a trapezoidal retaining wall ABCD retaining the earth upto a height 'h' on the vertical face AD. Let the earth surface is horizontal & it is in level with the top of the retaining wall.

Let AE is the rupture plane which means if the wall AD is removed the wedge AED of earth will move down along the plane AE. Let 'p' is the horizontal force offered by the retaining wall, to keep the wedge AED in equilibrium.

Let w is the weight density of the earth or soil.

Consider one metre length of the retaining wall

The force acting on the wedge AED of the retained material are:

(i) Weight of wedge AED.

W = weight density of earth × Area of AED × 1

$$= w \times \frac{AB \times ED}{2} \times 1$$

$$= w \times \frac{h \times h\cot\theta}{2} \quad \left(\because \tan\theta = \frac{AD}{ED} \text{ or } ED = \frac{AD}{\tan\theta} = h \times \cot\theta\right)$$

$$\boxed{W = \frac{w \times h^2 \times \cot\theta}{2}}$$

(ii) The horizontal force P exerted by the retaining wall on the wedge

(iii) The resultant reaction R at the plane AE. The reaction R is the resultant of normal reaction R and force of friction μR. The Resultant reaction R' makes an angle ϕ with the normal of the plane AE.

(iv) The friction resistance along the contact face AD is neglected.

These forces are similar as ~~Previes~~ Previous. fig- 2(a).

The wedge AED is in equilibrium under the action of three forces P, W & R'. The value of horizontal force P is given by equation (1). as

$$P = W\tan(\theta - \phi) \longrightarrow (1)$$

But here W = weight of wedge AED

$$= \frac{wh^2}{2}\cot\theta$$

Substituting the value of W in eqn ①

$$P = \frac{wh^2}{2} \cot\theta . \tan(\theta-\phi) \rightarrow ②$$

In the above equation the angle θ is the angle of the rupture plane. The earth is having maximum tendency to slip along rupture plane. Hence the supporting force 'P' should be maximum. But 'P' will be maximum if $\frac{dP}{d\theta} = 0$.

Hence differentiating eqn (ii) w.r. to θ we get.

$$\frac{dP}{d\theta} = \frac{d}{d\theta}\left[\frac{wh^2}{2}\cot\theta . \tan(\theta-\phi)\right] = 0$$

(or) $\frac{wh^2}{2}\left[\cot\theta \sec^2(\theta-\phi) - \csc^2\theta . \tan(\theta-\phi)\right] = 0$

(or) $\cot\theta \sec^2(\theta-\phi) - \csc^2\theta . \tan(\theta-\phi) = 0 \rightarrow ③$

Let $\tan\theta = t$ and $\tan(\theta-\phi) = t_1$.

The eqn ③ become as

$$\frac{1}{t}(1+t_1^2) - \left(1-\frac{1}{t^2}\right) \times t_1 = 0 \quad \left\{\because \cot\theta = \frac{1}{\tan\theta} = 1/t \; \& \; \csc^2\theta = 1+\cot^2\theta = 1+\frac{1}{t^2}\right\}$$

(or) $\frac{1+t_1^2}{t} - \left(\frac{t^2+1}{t^2}\right) \times t_1 = 0$

$t(1+t_1^2) - (t^2+1) \times t_1 = 0$

$t + tt_1^2 - t_1t^2 - t_1 = 0$

$t - t_1t^2 + tt_1^2 - t_1 = 0$

$t[1-t_1t] - t_1[1-tt_1] = 0$

(or) $(1-t_1t)(t-t_1) = 0$

either $(1-t_1t) = 0$ (or) $(t-t_1) = 0$

$\therefore tt_1 = 1$ (or) $t = t_1$

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If $t = t_1$, then $\theta = \tan(\theta - \phi)$

This is not possible.

$$\therefore tt_1 = 1$$

$$\tan\theta \tan(\theta - \phi) = 1$$

$$\therefore \tan\theta = \frac{1}{\tan(\theta-\phi)}$$

$$= \cot(\theta - \phi) = \tan[90 - (\theta - \phi)]$$

$$\theta = 90 - (\theta - \phi)$$

$$\theta + \theta - \phi = 90^0$$

$$2\theta - \phi = 90^0$$

$$\boxed{\theta = \frac{90 + \phi}{2} = 45^0 + \frac{\phi}{2}}$$

Thus plane of rupture is inclined at $(45^0 + \phi/2)$ ~~with~~ with the horizontal. Substituting the value of θ in eqn (2), we get

$$P = \frac{wh^2}{2}\cot\theta . \tan(\theta - \phi) = \frac{wh^2}{2} - \frac{\tan(\theta-\phi)}{\tan\theta}$$

$$= \frac{wh^2}{2} - \frac{\tan(45^0 + \phi/2 - \phi)}{\tan(45^0 + \phi/2)}$$

$$= \frac{wh^2}{2} - \frac{\tan(45^0 - \phi/2)}{\tan(45^0 + \phi/2)}$$

$$= \frac{wh^2}{2}\left[\frac{\tan 45^0 - \tan\phi/2}{1 + \tan 45^0 \tan\phi/2}\right] \times \left[\frac{1 - \tan 45^0 . \tan\phi/2}{\tan 45^0 + \tan\phi/2}\right]$$

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$$= \frac{wh^2}{2}\left(\frac{1-\tan\phi/2}{1+\tan\phi/2}\right)\times\left(\frac{1-\tan\phi/2}{1+\tan\phi/2}\right).$$

$$= \frac{wh^2}{2}\left(\frac{1-\tan\phi/2}{1+\tan\phi/2}\right)^2 = \frac{wh^2}{2}\left\{\frac{\left(1-\frac{\sin\phi/2}{\cos\phi/2}\right)}{\left(1+\frac{\sin\phi/2}{\cos\phi/2}\right)}\right\}^2$$

$$= \frac{wh^2}{2}\left\{\frac{\cos\phi/2-\sin\phi/2}{\cos\phi/2+\sin\phi/2}\right\}^2$$

$$= \frac{wh^2}{2}\left\{\frac{\cos^2\phi/2+\sin^2\phi/2-2\cos\phi/2\sin\phi/2}{\cos^2\phi/2+\sin^2\phi/2+2\cos\phi/2\sin\phi/2}\right\}$$

$$= \frac{wh^2}{2}\left\{\frac{1-2\cos\phi/2\sin\phi/2}{1+2\cos\phi/2\sin\phi/2}\right\}$$

$$\boxed{p = \frac{wh^2}{2}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]}$$

But p is the horizontal force exerted by the retaining wall on the wedge. The wedge of the earth will also exert the same horizontal force on the retaining wall. Hence above eqn. gives the horizontal force exerted by the earth on the retaining wall.

The horizontal force 'p' acts at a height of $h/3$ above the base.

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The Pressure intensity at the bottom.

If we assume a linear variation of the Pressure intensity varying from zero at the top to the maximum value 'p' at the bottom, then we have

$$P = \frac{p \times h}{2}$$

$$P = \frac{wh^2}{2}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]$$

equating the two values of P, we get

$$\therefore \frac{p \times h}{2} = \frac{wh^2}{2}\left[\frac{1-\sin\phi}{1+\sin\phi}\right]$$

$$\boxed{p = wh\left[\frac{1-\sin\phi}{1+\sin\phi}\right]}$$

(1) A masonry retaining wall of trapezoidal section is 6m high & retains earth which is level upto the top. The width at the top is 1m and the exposed face is vertical. Find the minimum width of the wall at the bottom in order the tension may not be induced at the base. The density of masonry and earth is 2300 & 1600 kg/m³ respectively. The angle of repose of soil is 30°.

Sol^n. Height of wall = 6m

Top width = 1m

Density of masonry = ρ_0 = 2300 kg/m³

∴ weight density of masonry

$$w_0 = \rho_0 \times g = 2300 \times 9.81 \ N/m^3$$

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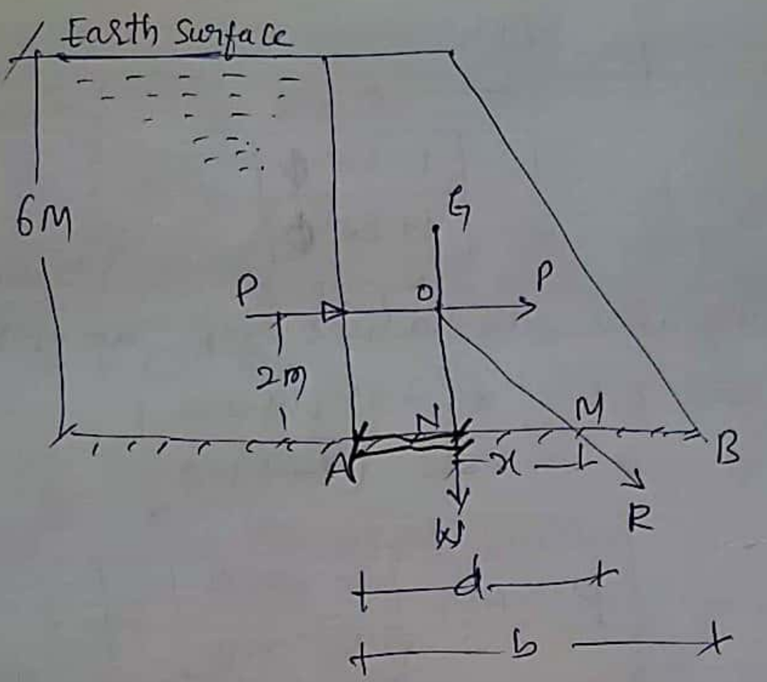
Density of earth, $\rho = 1600\ kg/m^3$

$\therefore$ weight density of earth, $w = \rho \times g = 1600 \times 9.81\ N/m^3$

$\therefore$ Angle of repose, $\phi = 30^\circ$

Let b = minimum width ~~of~~ at the bottom



Consider one metre length of the retaining wall

$$P = \frac{1}{2} w h^2 \left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

$$= \frac{1}{2} \times 1600 \times 9.81 \times 6^2 \times \left(\frac{1-\sin 30^\circ}{1+\sin 30^\circ}\right)$$

$$= 800 \times 9.81 \times 36 \cdot \left(\frac{1-0.5}{1+0.5}\right) = \frac{800 \times 9.81 \times 36 \times 0.5}{1.5}$$

$$\boxed{P = 94176 N}$$

The thrust P will acting at a height of $6/3 = 2m$ above the base. weight of 1m length of trapezoidal wall,

W = weight density of masonry × Area of trapezoidal × 1

$$= 2300 \times 9.81 \times \left(\frac{a+b}{2}\right) \times h \times 1$$

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$$= 2300 \times 9.81 \times \left(\frac{1+b}{2}\right) \times 6 = \underline{67689(1+b)\,N}.$$

The weight W will be acting through the C.G of the trapezoidal section. The distance of the C.G. of trapezoidal from the point A. is obtained by the equation.

$$\therefore \boxed{AN = \frac{a^2+ab+b^2}{3(a+b)} = \frac{1^2+1\times b+b^2}{3(1+b)} = \frac{1+b+b^2}{3(1+b)}}$$

The horizontal distance 'x'

$$x = P/W \times h/3$$

$$= \frac{94176}{67689(1+b)} \times 6/3 = \frac{2.272}{1+b}$$

$$\boxed{x = \frac{2.272}{1+b}}$$

$$d = AN + x$$

$$= \frac{1+b+b^2}{3(1+b)} + \frac{2.272}{1+b}$$

$$= \frac{1+b^2+b^2+3\times 2.272}{3(1+b)} = \frac{1+b+b^2+8.346}{3(1+b)}$$

$$\boxed{d = \frac{b^2+b+9.346}{3(1+b)}}$$

If the tension at the base is just avoided

$$d = 2/3\,b$$

$$\frac{b^2+b+9.346}{3(1+b)} = 2/3\,b$$

$b^2 + b + 9.346 = 2b(1+b) = 2b + 2b^2$

$$\boxed{b^2 + b - 9.346 = 0}$$

$$\therefore \boxed{b = 2.597\,m}$$

(2) A masonry retaining wall of trapezoidal section is 10m high and retains earth which is level upto the top. The width at the Top is 2m and the bottom is 8m & the exposed face is vertical. Find the maximum and minimum intensities of normal stress on the base. Take, Density of earth = 1600 kg/m³

Density of masonry is 2400 kg/m³.

Angle of repose of earth = 30°.

Density of earth $\rho = 1600\ kg/m^3$

$\therefore$ weight density of earth

$w = \rho \times g = 1600 \times 9.81\ N/m^3$

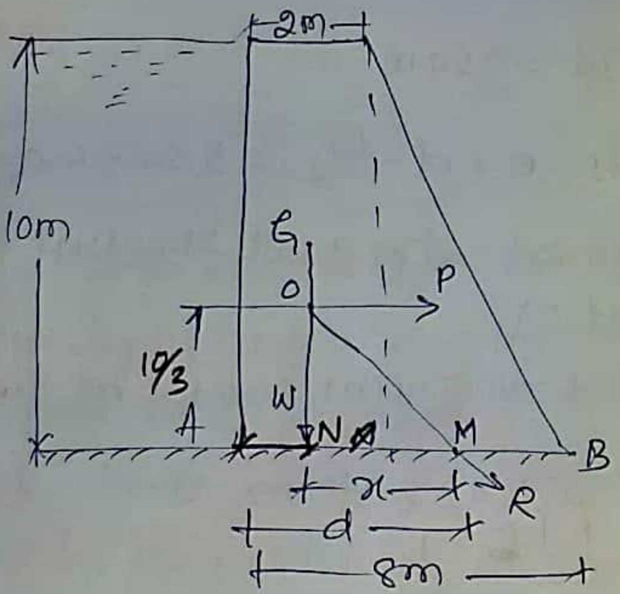
Density masonry. $\rho_0 = 2400\ kg/m^3$

$\therefore$ weight density of masonry

$w_0 = \rho_0 \times g = 2400 \times 9.81\ N/m^3$

Angle of repose $\phi = 30^\circ$

Consider 1m length of the wall.



Thrust of earth on the vertical face of the wall is given by equation

$$P = \frac{1}{2} w h^2 \left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

$$= \frac{1}{2} \times 1600 \times 9.81 \times 10^2 \left(\frac{1-\sin 30}{1+\sin 30^\circ}\right)$$

$$\boxed{P = \frac{80000 \times 9.81}{3} N}$$

The thrust P will be acting at a height of $10/3$ m above the ground. weight of 1m length of trapezoidal wall.

W = weight density of masonry x volume of wall

$$= 2400 \times 9.81 \times \left(\frac{2+8}{2}\right) \times 10 \times 1$$

$$\boxed{W = 120000 \times 9.81 N}$$

∴ Find 'x' value

$$x = P/W \times h/3 = \frac{80000 \times 9.81}{3 \times 120000 \times 9.81} \times \frac{10}{3} = 0.74 m$$

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$$d = AN + x$$
$$= 2.8 + 0.74 = \underline{\underline{3.54\,m}}$$

$\therefore$ Eccentricity $e = d - b/2 = 3.54 - 4.0 = \underline{\underline{-0.46\,m}}$

(minus sign only indicates that stress at A will be more than at B).

The maximum & minimum stresses at the base are given by.

$$\sigma_{max} = \frac{W}{b}\left(1 \pm \frac{6.e}{b}\right)$$

$$= \frac{120000 \times 9.81}{8}\left(1 \pm \frac{6 \times 0.46}{8}\right)$$

$$\boxed{\sigma_{max} = 197916.75\ N/m^2} \ \&$$

$$\sigma_{min} = \frac{120000 \times 9.81}{8}\left(1 - \frac{6 \times 0.46}{8}\right)$$

$$\boxed{\sigma_{min} = 96383.25\ N/m^2}$$

(3) A masonry retaining wall of trapezoidal section is 1.5m wide at the top 3.5m wide at the base and 6m high. The face of the wall retaining earth is ~~1.5m wide at the~~ vertical and the earth level is upto the top of the wall. The density of the earth is ~~vertical and the~~ 1600 kg/m^3 for the top 3m and 1800 kg/m^3 below this level. The density of masonry is 2300 kg/m^3. Find the total lateral Pressure on the retaining wall per m run and maximum and minimum normal Pressure intensities at the base. Take the angle of repose = 30° for both type of earth.

Soln :-

$a = 1.5m$

$b = 3.5m$

$h = 6m$

Density of upper earth. $\rho_1 = 1600\ kg/m^3$

weight density of earth $w_1 = 1600 \times 9.81\ N/m^3$.

Depth of upper earth $h_1 = 3m$

Density of lower earth $\rho_2 = 1800\ kg/m^3$

weight density of earth $w_2 = 1800 \times 9.81\ N/m^3$

Depth of lower earth $h_2 = 3m$

Density of masonry $\rho_0 = 2300\ kg/m^3$

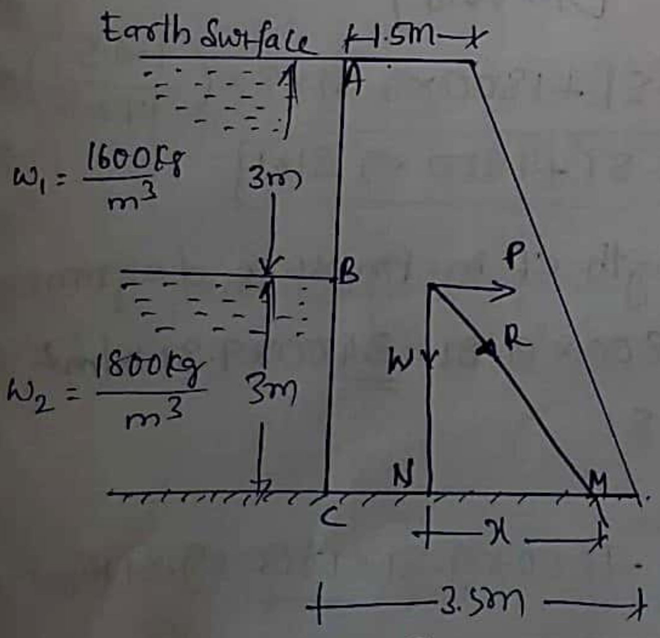
$\therefore$ weight density of masonry.

$w_0 = 2300 \times 9.81\ N/m^3$.

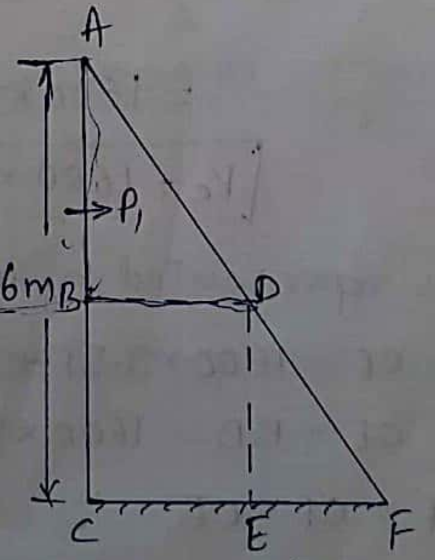
Angle of repose for both earth

$\phi = 30^\circ$

Total lateral Pressure on the retaining wall per m run.



(a)



Pressure diagram

(b)

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The Pressure diagram on the retaining wall is

let P = Total lateral pressure force

P_1 = Pressure force due to upper earth

P_2 = Pressure force due to lower earth

The pressure intensity at a depth 'h' is given by eqn.

$$P = wh\left[\frac{1-\sin\phi}{1+\sin\phi}\right]$$

∴ Pressure intensity at B.

$$P_B = w_1 h_1\left(\frac{1-\sin\phi}{1+\sin\phi}\right) = 1600 \times 9.81 \times 3\left(\frac{1-0.5}{1+0.5}\right)$$

$$= 4800 \times 9.81 \times \frac{0.5}{1.5} = \underline{1600 \times 9.81\ N/m^2}$$

this is represented by length BD in Pressure diagram

∴ length $BD = P_B = 1600 \times 9.81\ N/m^3$

Similarly Pressure intensity at C.

$$P_C = P_B + w_2 h_2\left[\frac{1-\sin\phi}{1+\sin\phi}\right]$$

$$= 1600 \times 9.81 + 1800 \times 9.81 \times 3 \times \left(\frac{1-0.5}{1+0.5}\right)$$

$$\boxed{P_C = 1600 \times 9.81 + 1800 \times 9.81 N}$$

this is represented by length CF in Pressure diagram.

$\therefore CF = 1600 \times 9.81 + 1800 \times 9.81 = \underline{3400 \times 9.81\ N/m^2}$

$CE = BD = 1600 \times 9.81$

$\therefore EF = CF - CE$

$= (1600 + 1800) \times 9.81 - 1600 \times 9.81 = \underline{1800 \times 9.81\ N/m^2}$

∴ ~~thus is~~ Pressure force due upper earth.

$$P_1 = \text{Area of } \Delta^{le} ABD.$$
$$= \frac{1}{2} \times AB \times BD = \frac{1}{2} \times 3 \times 1600 \times 9.81 = \underline{23544N}$$

This force acts at a height of $\frac{1}{3} \times 3 = 1m$ above 'B' or at a height of $(3+1) = 4m$ above. point C.

Pressure force due to lower earth.

$$P_2 = \text{Area of } BDFC = \tfrac{1}{2}\left[BD + CF\right] \times BC$$
$$= \frac{1}{2}\left[1600 + 3400\right] \times 9.81 \times 3.0$$
$$\boxed{P_2 = 73575N}$$

This force acts at a height from C.

$$= \Big[\text{Area of rectangle } CEDB \times 3/2$$
$$+ \text{Area of } \Delta^{le} EFD \times 1\Big] \div \text{Total area}$$
$$= \frac{1600 \times 9.81 \times 3 \times 3/2 + \frac{1800 \times 9.81 \times 3}{2} \times 1}{1600 \times 9.81 \times 3 + \frac{1800 \times 9.81 \times 3}{2}}$$
$$= \frac{9.81 \times 7200 + 2700 \times 9.81}{9.81 \times 4800 + 2700 \times 9.81} = \frac{9900}{7500} = 1.32m \text{ from. C.}$$

∴ Total Pressure force,

$$P = P_1 + P_2 = 23544 + 73575 = \underline{97119N.}$$

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Maximum and minimum normal stresses at base

Weight of retaining wall per m run.

W = weight density of masonry $\times \left(\frac{a+b}{2}\right) \times h \times 1$

$= 2300 \times 9.81 \times \left(\frac{1.5+3.5}{2}\right) \times 6 \times 1 = 338445 N.$

The weight W will be acting at the C.G of the retaining wall. The distance of the C.G of the retaining wall from point C is given.

$$CN = \frac{a^2+ab+b^2}{3(a+b)} = \frac{1.5^2+1.5\times3.5+3.5^2}{3(1.5+3.5)} = 1.32m$$

Let x = Distance b/w the line of action of W & the resultant of W & P at the base.

Taking moments of W, P_1 & P_2 about point M, we get

$$P_1 \times 4 + P_2 \times 1.32 = W \times x$$

$$x = \frac{P_1 \times 4 + P_2 \times 1.32}{W}$$

$$= \frac{23544 \times 4 + 73575 \times 1.32}{338445} = \frac{94176+97119}{338445} = 0.565m$$

$\therefore$ eccentricity $e = CM - b/2 = 1.885 - \frac{3.5}{2} = 0.135m$

$\therefore$ Distance $CM = CN + x = 1.32 + 0.565 = 1.885m$

max & min stresses are:-

$$\sigma_{max} = \frac{W}{b}\left(1+\frac{6 \cdot e}{b}\right) = \frac{338445}{3.5}\left(1+\frac{6\times0.135}{3.5}\right)$$

$$= 119073.78 N/m^2$$

and

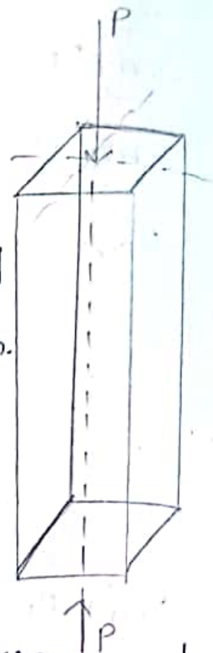
$$\sigma_{min} = \frac{W}{b}\left(1-\frac{6\times e}{b}\right) = \frac{338445}{3.5}\left(1-\frac{6\times0.135}{3.5}\right)$$

$$= 74320.56 N/m^2$$

19/02/2020

UNIT-3

COLUMNS



Column :-

- It is a vertical member of a structure subjected to vertical compressive load is called column.
- columns are subjected to only vertical compressive load.

struts :-

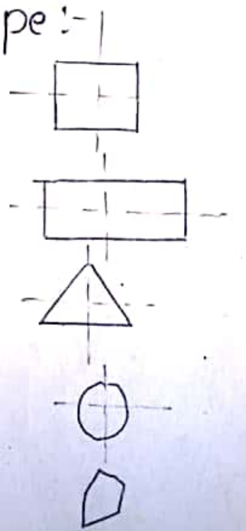
- It is a comparatively short column and it may be horizontal, vertical (or) inclined.
- struts are used in frames, 8th trusses etc.

Classification of column :-

(1) Based on shape.
(2) Based on height
(3) Based on load → (a) Axial b) uniaxial (c) Bi-axial
(4) Based on failure of column.
(a) crippling (b) crushing

(1) Based on shape :-

(a) square
(b) Rectangular
(c) Triangular
(d) circular
(e) polygonal



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(2) Based on height :-

(1) long column

(2) Medium column

(3) short column

(1) long column :-

It is that column in which the effective length to the least lateral dimension is greater than 12 is called "long column".

$$\boxed{\frac{le}{D} > 12}$$

→ If the column is long, then it will fail only because of buckling (or) crippling

→ In long columns, direct stresses are very small compared to their buckling stresses

→ long column is a column whose slenderness ratio is greater than 120.

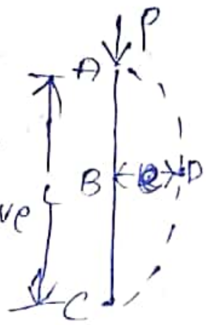
$$\boxed{\frac{le}{K} > 120}$$

→ whose length is more than 30 times the least (lateral dimension). (30 D)
radius

→ For mild steel column slenderness ratio is >80

$$\boxed{\frac{le}{K} > 80}$$

failure of a long column :-



→ A long column uniform c/s area A and length (L), subjected to an axial compressive load 'p' when a column is known as long column.

→ when applying a compressive load on the column the column will bending (buckling).

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Short column:-

It is that column in which the effective length to the least lateral dimension is less than 12.

$$\frac{l_e}{D} < 12 \rightarrow \text{short column}$$

⇒ short column whose slenderness ratio is

$$\frac{l_e}{K} < 32$$

whose length is less than 8 times the lateral dimension.

$$L < 8D$$

⇒ For mild steel column slenderness ratio is less than 8D.

⇒ If the column in short, than it will fail only because of direct stress. (compressive).

Medium column:-

→ B/w 12 is called Medium column

→ For medium column, the slenderness ratio is more than 32 and less than 120.

→ less than 30D and >8D

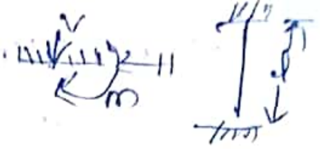
(*) Types of end conditions of columns:-

1. Fixed end
2. Pinned end (or) hinged end
3. Free end.

(1) Fixed end:-

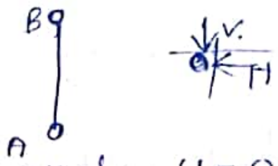
In this case the end is fixed both in position and direction. Deflection and slop is zero at fixed end.

$y=0, \quad \frac{dy}{dx}=0$



(2) pinned end (or) hinged end :-

In this case the end is fixed in position only.



→ deflection is zero at hinged end. $y=0$

(3) Free end :-

In this case, the end of the column is free.

End conditions of long column :-

1. Both ends are hinged $l = l_e$
2. Both ends are fixed $l_e = \frac{l}{2}$
3. one end is fixed and other end is hinged. $\Rightarrow l_e = \frac{l}{\sqrt{2}}$
4. one end is fixed and other end is free $\Rightarrow l_e = 2l$

Radius of gyration (R) (or) (K) :-

The ratio square root of moment of Inertia (I) to the cross-sectional area 'A' is called "radius of gyration".

$K = \sqrt{\frac{I}{A}}$ where, I - moment of Inertia in mm^4

A - Area of c/s mm^2

K - radius of gyration in mm

Slenderness ratio :-

The ratio of effective length to least radius of gyration is called "slend. ratio"

slenderness ratio $= \frac{l}{K_{min}}$ (or) $\frac{l}{r_{min}}$

→ The load carrying capacity of long column is depends on slenderness ratio.

Buckling load :-

The load acts which the column just buckles is called "Buckling load".

→ Buckling load is load for long column.

Factor of safety:-

$$F.O.S = \frac{\text{Crippling load}}{\text{Safe load}}$$

Column subjected to axial load:-

Euler's crippling load ⇒ $P_{cr} = \frac{\pi^2 EI}{l_e^2}$

where

EI = flexural rigidity

L = effective length of the column

→ Eulers formula is need for long column.

Assumptions made in Euler's formula:

1. the column is initially perfectly straight and load is applied axially.
2. The c/s section of the column is uniform throught out its length.
3. the length of the column is very large compared to the lateral dimen-sions.
4. The self weight of the column is ignorable.
5. The column fails due to buckling alone.
6. The column material is perflecting elaster, homogeneous, Isotropic and Obeys the hook's law.

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Euler's theory :-

For crippling load

$$P = \frac{\pi^2 E I}{l^2}$$

where,

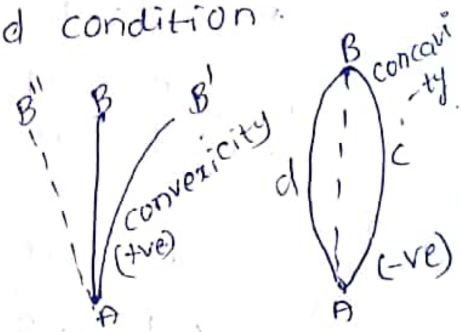
P = crippling load

E = modulus of elasticity for the column material

I = least moment of Inertia of column section.

l = equivalent length of column for the given end condition.

Sign conversions:-



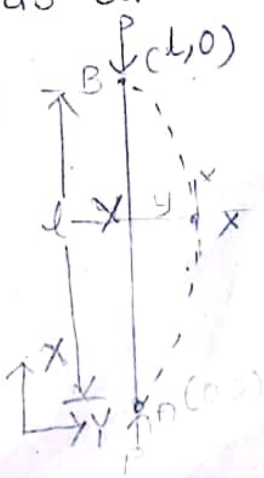
where AB - Initial central line

⇒ when at moment which will bend the column with its convexity towards its central line taking as '+ve'

2. A moment which will tend to bend the column with its concavity towards its initial centre line. Taking as "-ve".

20/02/2020

Expression for column both ends are subjected to hinge (or) pin jointed.



$$EI \frac{d^2y}{dx^2} = -M \longrightarrow (1)$$

M at y-y ⇒ P×y.

$$\boxed{M = Py}$$

$$EI \frac{d^2y}{dx^2} = -Py$$

$$\frac{d^2y}{dx^2} = \frac{-Py}{EI}$$

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$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0 \longrightarrow (2)$$

$$\frac{P}{EI} = k^2 \longrightarrow (3)$$

From (2) & (3)

$$\frac{d^2y}{dx^2} + yk^2 = 0$$

$$\boxed{y = C_1\cos(kx) + C_2\sin(kx)} \longrightarrow (4)$$

Boundary conditions -

$x=0, y=0$

sub. in eq(4)

$$0 = C_1 + 0$$

$$\boxed{\therefore C_1 = 0}$$

$$y_2 = 0 + C_2\sin(kx)$$

$$x = l, y = 0$$

$$0 = C_2\sin(kl)$$

$$\sin kl = 0$$

$$kl = n\pi \quad [0, \pi, 2\pi, 3\pi]$$

$$k = \frac{n\pi}{l}$$

Squaring on both sides

$$k^2 = \frac{n^2\pi^2}{l^2}$$

$$\therefore \frac{P}{EI} = \frac{n^2\pi^2}{l^2} \quad \left[\because k^2 = \frac{P}{EI}\right]$$

$$\boxed{\therefore P = \frac{n^2\pi^2 EI}{l^2}} \quad (n\pi = 0, \pi, 2\pi)$$

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$$\boxed{\therefore P = \frac{\pi^2 EI}{l^2}}$$

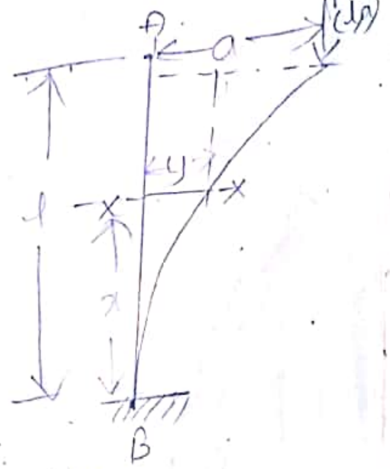
$P \propto EI$

$P \propto \frac{1}{l^2}$

$n =$ mode member

(B) The column one end is fixed and another end is free:- having length (l)

The column one end is fixed and another end is free having length (l), The load acting at point at a distance 'a' and distance (or) diffelection at a distance of y at x-axis.



$$EI \frac{d^2y}{dx^2} = P(a-y)$$

$$EI \frac{d^2y}{dx^2} = Pa - Py$$

$$EI \frac{d^2y}{dx^2} + Py = Pa$$

Divide with EI on both sides

$$\boxed{\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{Pa}{EI}} \qquad \frac{P}{EI} = k^2$$

$$y = C_1 \cos kx + C_2 \sin kx + a \longrightarrow (1)$$

$x = 0,\ y = 0$ sub in eq (1)

$$0 = C_1(1) + C_2(0) + a$$

$$\boxed{C_1 = -a}$$

$y = C_1\cos(kx) + C_2\sin kx + a$

$\frac{dy}{dx} = C_1(-\sin(kx))k + C_2[\cos(kx)]k + 0$

$x = 0,\ \frac{dy}{dx} = 0$ sub in eq (1)

$0 = C_1(0) + C_2(k)$

$kC_2 = 0$

$$\boxed{C_2 = 0}$$

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from eq(1), c_1 & c_2

$x = l, y = a$ $\quad$ $x = l, y = a$

$$a = -a\cos(kl) + (0)\sin kx + a$$

$$\cancel{a} = -a\cos kl + \cancel{a}$$

$$a\cos kl = 0$$

$$kl = 0, \frac{\pi}{2}$$

$$kl = \pi/2$$

Squaring on both sides $\quad k^2 l^2 = \pi^2/4$

$$\frac{P}{EI} l^2 = \frac{\pi^2}{4} \qquad \left[\because k^2 = \frac{P}{EI}\right]$$

$$\boxed{\therefore P = \frac{\pi^2 EI}{4l^2}}$$

1) A mild steel tube 4m long, 30mm internal dia. and 4mm externa thick is used as a strut with both ends hinged. Find the collapsing load. Take $E = 2.1 \times 10^5 \ N/mm^2$

Given.

$L = 4\,m \Rightarrow 4000\,mm$

$d = 30\,mm$

$t = 2mm, D = (4+4+30)\,mm \Rightarrow 38\,mm$

Both ends fixed. $l = l_{eff}$

$$I = \frac{\pi(D^4 - d^4)}{64} \Rightarrow \frac{\pi(38^4 - 30^4)}{64} \Rightarrow 62559.309\ mm^4$$

$E = 2.1 \times 10^5 \ N/mm^2$

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$$P = \frac{\pi^2 EI}{l^2} \Rightarrow \frac{\pi^2 \times 2.1\times10^5 \times 62593.09}{(4000)^2}$$

$$P = 8108.21\ N$$

$$\boxed{\therefore P = 8.108\ KN}$$

2) $P = 10KN$, find 'l':

$$P = \frac{\pi^2 EI}{l^2}$$

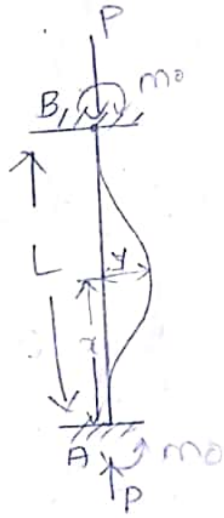
$$l = \sqrt{\frac{\pi^2 EI}{P}} \Rightarrow \sqrt{\frac{\pi^2 \times 2.1\times10^5 \times 62593.09}{10\times1000}}$$

$$l = 3601.82\ mm$$

$$\boxed{\therefore l = 3.601 m}$$

4/03/2020

case-III: Both ends are fixed.



$$M = m_0 - Py$$

$$EI \frac{d^2y}{dx^2} = M$$

$$EI \frac{d^2y}{dx^2} = m_0 - Py$$

$$EI \frac{d^2y}{dx^2} + Py = m_0$$

divide with EI on both sides, we get

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_0}{EI}$$

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{M_0}{EI} \times \frac{P}{P}$$

$$\left[\because \frac{P}{EI} = K^2\right]$$

$$\frac{d^2y}{dx^2} + K^2 y = K^2 \frac{m_0}{P}$$

$$K = \sqrt{P/EI}$$

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$$y = C_1 \cos(xk) + C_2 \sin(xk) + \frac{m_0}{P} \longrightarrow (1)$$

Differentiating on both sides with "x"

$$\frac{dy}{dx} = (-1) C_1 \sin(xk) k + C_2 \cos(kx) k + 0 \longrightarrow (2)$$

$x = 0$, $\frac{dy}{dx} = 0$ put eq (2)

$$0 = (-1) C_1 \sin(0) + C_2 \cos(0) k + 0$$

$$\boxed{\therefore C_2 = 0}$$

$x = 0, y = 0$ put eq (1)

$$0 = C_1 \cos(0) + C_2 \sin(0) + \frac{m_0}{P}$$

$$\boxed{C_1 = -\frac{m_0}{P}}$$

$x = l, y = 0$ put eq (1)

$$0 = -\frac{m_0}{P} \cos(lk) + (0) \sin(lk) + \frac{m_0}{P}$$

$$0 = +\frac{m_0}{P} [1 - \cos(lk)]$$

$$0 = 1 - \cos(lk)$$

$$\cos(lk) = 1$$

$$\cos(lk) = \cos(2\pi)$$

$$lk = 2\pi$$

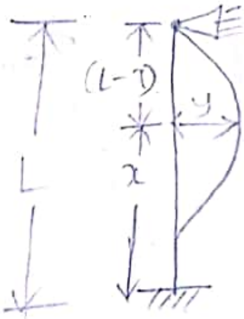
$$l \sqrt{\frac{P}{EI}} = 2\pi$$

Squaring on both sides

$$l^2 \cdot \frac{P}{EI} = (2)^2(\pi)^2$$

$$\boxed{\therefore P = \frac{4\pi^2 EI}{l^2}}$$

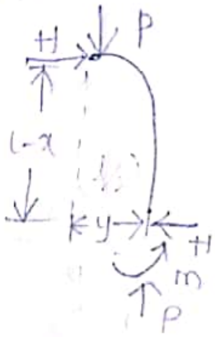
05/03/2020 column with one end is fixed and other end is hinged.



$$M = -Py + H(L-x)$$

$$EI\frac{d^2y}{dx^2} = -Py + H(L-x)$$

$$EI\frac{d^2y}{dx^2} + Py = H(L-x)$$



Divide both sides with EI.

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{H(L-x)}{EI}$$

multiply Divide with 'P' on Right side.

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{P \cdot H(L-x)}{PEI}$$

$$\frac{P}{EI} = k^2$$

$$\frac{d^2y}{dx^2} + k^2y = \frac{k^2 H(L-x)}{P}$$

Solving the eq^n w.r.t. 'x' on both sides,

$$y = C_1\cos(kx) + C_2\sin(kx) + \frac{H(L-x)}{P} \rightarrow (1)$$

Differen. eq(1) w.r.t "x"

$$\frac{dy}{dx} = C_1\frac{d}{dx}\cos(kx) + C_2\frac{d}{dx}\sin(kx) + \frac{d}{dx}\frac{H(L-x)}{P} \rightarrow (2)$$

$$= -C_1\sin(kx)k + C_2\cos(kx)\cdot k + \frac{H}{P}(-1)$$

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$\Rightarrow -C_1 k \sin(kx) + C_2 k \cos(kx) - \frac{H}{P}$

$x=0, \frac{dy}{dx}=0$

$0 = 0 + k C_2 (1) - \frac{H}{P}$

$\boxed{C_2 = \frac{H}{Pk}}$

$x=0, y=0$ in eq(1)

$0 = C_1 + 0 + \frac{H}{P}(l)$

$\boxed{C_1 = -\frac{Hl}{P}}$

$x=l, y=0$, in eq(1)

$0 = C_1 \cos(kl) + C_2 \sin(kl) + \frac{H}{P}(l-l)$

$0 = -\frac{Hl}{P}\cos(kl) + \frac{H}{Pk}\sin(kl)$

$0 = \frac{H}{P}\left(\frac{\sin kl}{k} - l\cos kl\right)$

$\frac{\sin kl}{k} = l\cos kl$

$\frac{\sin kl}{\cos kl} = kl$

$\tan kl = kl$

$l = 4.5$

Squaring on both sides

$k^2 l^2 = 4.5^2$

$k^2 l^2 = 20.25$

$\frac{P}{EI} l^2 = 20.25$

$P = \frac{20.25\, EI}{l^2}$

$\boxed{\because 2\pi^2 = 20.25}$

$\boxed{\therefore P = \frac{2\pi^2 EI}{l^2}}$

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Sl.No.	condition	crippling load Actual length	crippling load Effective length	Relation b/w effective length & AL
1.	Both ends are hinged	$\frac{\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l^2}$	$l_e = l$
2.	one end is fixed other is free	$\frac{\pi^2 EI}{4l^2}$	$\frac{\pi^2 EI}{l^2}$	$l_e = 2l$
3.	One end is fixed other is hinged	$\frac{2\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l^2}$	$l_e = \frac{l}{\sqrt{2}}$
4.	Both ends are fixed	$\frac{4\pi^2 EI}{l^2}$	$\frac{\pi^2 EI}{l^2}$	$l_e = \frac{1}{2} l$

The length of the solid bar is 3.5m and having dia of 8 cm used as a strut take $E = 2.1 \times 10^5 N/mm^2$, calculate crippling load (collapse load) for different end conditions.

$l = 3.5m \Rightarrow 3500mm$

$d = 8cm \Rightarrow 80mm$

$I = \frac{\pi d^4}{64} \Rightarrow 2.01 \times 10^6 mm^4$

(i) Both ends are hinged

$$P = \frac{\pi^2 EI}{l^2} \Rightarrow \frac{\pi^2 \times 2.1 \times 10^5 \times 2.01 \times 10^6}{3500^2}$$

$[l_e = l]$

$$\boxed{P = 340.07 \times 10^3 N}$$

(ii) Both ends are fixed

$$l_e = \frac{l}{2}$$

$$P = \frac{4\pi^2 EI}{l^2} \Rightarrow \frac{4\pi^2 (2.1\times10^5) \times (2.01\times10^6)}{3500^2}$$

$$\boxed{P = 1360.31\times10^3 N}$$

(iii) one end is fixed other is free

$$l_e = 2l$$

$$P = \frac{\pi^2 EI}{4l^2} \Rightarrow \frac{\pi^2 \times 2.1\times10^5 \times 2.01\times10^6}{3500^2 \times 4}$$

$$\boxed{\therefore P = 85.01\times10^3 N}$$

(iv) one end is fixed other is hinged

$$l_e = \frac{l}{\sqrt{2}}$$

$$P = \frac{2\pi^2 EI}{l^2}$$

$$\Rightarrow \frac{2\pi^2 \times 2.1\times10^5 \times 2.01\times10^6}{3500^2}$$

$$\boxed{\therefore P = 680.01\times10^3 N}$$

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6/03/2020

A wooden column of size 15cm × 20cm and height of 6m with $E = 17.5 KN/mm^2$ Determine failure load for

(i) Both ends are hinged

(ii) one end is hinged and one end is fixed

(iii) Both ends are fixed.

(iv) One end is fixed and other is free.

$L = 6000 mm$

$b \times D = 15cm \times 20cm$

$I_{xx} = \frac{bxd^3}{12} \Rightarrow \frac{150 \times 200^3}{12}$

$\Rightarrow 100 \times 10^6 mm^4$

$I_{YY} \Rightarrow \frac{db^3}{12} \Rightarrow 56.25 \times 10^6 mm^4$

(i) $P = \frac{\pi^2 EI}{l^2}$

$le = l$

$P = \frac{\pi^2 \times 17.5 \times 10^3 \times 56.25 \times 10^6}{(6000)^2}$

$P = 269.87 \times 10^3 N$

$\boxed{\therefore P = 269.87 \times 10^3 N}$

(ii) $le = \frac{l}{\sqrt{2}}$

$P = \frac{\pi^2 EI}{(\frac{l}{\sqrt{2}})^2} \Rightarrow \frac{\pi^2 \times 17.5 \times 10^3 \times 56.25 \times 10^6}{(\frac{6000}{\sqrt{2}})^2}$

$\boxed{P = 539.74 KN}$

(iii) $le = l/2$ $P = \frac{\pi^2 EI}{(l/2)^2} \Rightarrow \frac{\pi^2 \times 17.5 \times 10^3 \times 56.25 \times 10^6}{(6000/2)^2}$

$\boxed{\therefore P = 1079 KN}$

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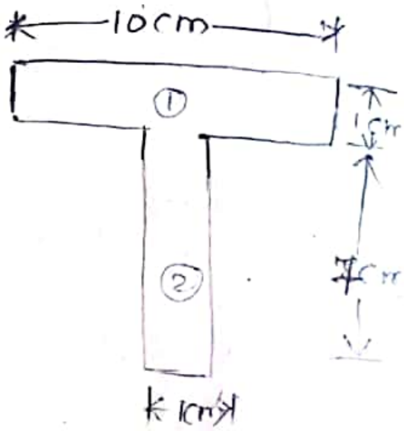
(iv) $l_e = 2l$

$$P = \frac{\pi^2 EI}{l^2} \Rightarrow \frac{\pi^2 EI}{(2l)^2} \Rightarrow \frac{\pi^2 \times 17.5\times10^3 \times 56.25\times10^{...}}{(2\times6000)^2}$$

$$\boxed{\therefore P = 67.46\ KN}$$

11/03/2020

(2)



T - 10cm × 8cm × 1cm

E - $2\times10^5\ N/mm^2$

L - 3m - 3000 mm

E.C - B.E.R.F

P = ?

$$P = \frac{\pi^2 EI}{le^2}$$

$$\bar{y}_{\text{from bottom}} = \frac{a_1y_1 + a_2y_2}{a_1+a_2}$$

$$I_{xx} = I_g + Ah_1^2$$

$$\bar{y} = \frac{10\times1[7+1/2] + (1\times7)[7/2]}{(10\times1)+(1\times7)}$$

$$I_{xx_1} = \frac{bd^3}{12} + A(\bar{y} - (7+\tfrac{1}{2}))^2$$

$\bar{y} = 5.85\ cm$ from bottom

$$\Rightarrow \frac{10\times1^3}{12} + (10\times1)(5.85 - 7.5)^2$$

$$\Rightarrow 28.058\ cm^4$$

$$I_{xx_2} \Rightarrow \frac{bd^3}{12} + A(h_2)^2 \Rightarrow \frac{1\times7^3}{12} + (1\times7)(5.85 - 7/2)^2$$

$$\Rightarrow 67.24\ cm^4$$

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$I_{xx} = I_{xx_1} + I_{xx_2} \Rightarrow 28.058 + 67.24 \Rightarrow 95.29\, cm^4$

$\Rightarrow 95.29 \times 10^4\, mm^4$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} \Rightarrow \frac{\frac{10}{2}(10 \times 1) + (1 \times 7)(4.5 + 0.5)}{(10 \times 1) + (1 \times 7)}$$

$\bar{x} \Rightarrow 5$ cm from left

$I_{yy} = I_g + A h_1^2$

$$I_{yy_1} \Rightarrow \frac{bd^3}{12} + A h_1^2 \Rightarrow \frac{1 \times 10^3}{12} + (1 \times 10)(5-5)$$

$\Rightarrow 83.33\, cm^4$

$$I_{yy_2} \Rightarrow \frac{bd^3}{12} + A h_1^2 \Rightarrow \frac{7 \times 1^3}{12} + (1 \times 7)(5-5)$$

$\Rightarrow 0.583\, cm^4$

$\therefore I_{yy} \Rightarrow 83.91 \times 10^4\, mm^4$

$$P = \frac{\pi^2 E I}{\left(\frac{3000}{2}\right)^2} \Rightarrow \frac{\pi^2 \times 2 \times 10^5 \times 83.91 \times 10^4}{\left(\frac{3000}{2}\right)^2}$$

$P \Rightarrow 736.14 \times 10^3\, N$

$\boxed{\therefore P = 736.14\, kN}$

(2) $10 \times 10 \times 2$ cm

$L = 5$

F.O.S $= 3$

B.E.R. hinged

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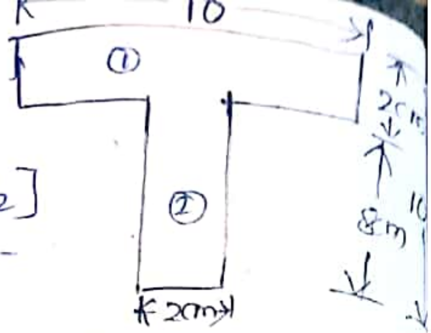
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$



$$\Rightarrow \frac{(10\times2)[8+2/2]+[8\times2][8/2]}{(10\times2)+(2\times8)}$$

$\Rightarrow$ 6.77 cm from bottom

$$I_{xx_1} = \frac{bd^3}{12} + Ah_1^2$$

$$\Rightarrow \frac{10\times2^3}{12} + (10\times2)\left[6.77 - (8+2/2)\right]^2$$

$\Rightarrow$ 106.124 cm^4

$$I_{xx_2} = \frac{bd^3}{12} + Ah_2^2 \Rightarrow \frac{2\times8^3}{12} + (8\times2)\left[6.77 - 8/2\right]^2$$

$\Rightarrow$ 208.099 cm^4

$I_{xx} = 314.22\, cm^4 \Rightarrow 314.223\times10^4\, mm^4$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} \Rightarrow 5\,cm \text{ from left}$$

$$I_{yy_1} = \frac{bd^3}{12} + Ah_1^2 \Rightarrow \frac{2\times10^3}{12} + (10\times2)(5-5)$$

$\Rightarrow$ 166.66

$$I_{yy_2} \Rightarrow \frac{8\times2^3}{12} + (2\times8)(5-5) \Rightarrow 0 + 5.33$$

$\Rightarrow 5.33$

$I_{yy} \Rightarrow 171.99\times10^4\, mm^4$

$$F.O.S = \frac{\text{Ultimate load}}{\text{working load}}$$

$$P = \frac{\pi^2 EI}{l_e^2} \Rightarrow \frac{\pi^2 \times 2\times10^5 \times 171.99\times10^4}{5000^2}$$

$$\Rightarrow 135797.86\ N$$

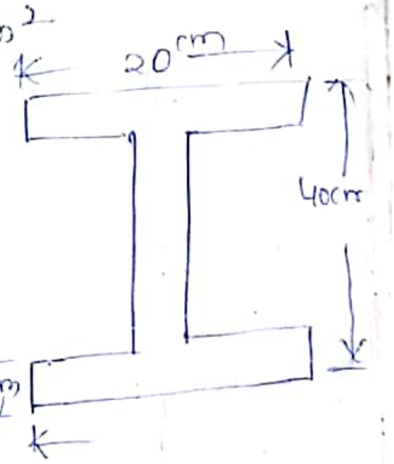
$$\Rightarrow 135.79\ kN$$

Ultimat load = F.O.S x working load

$$\Rightarrow 3 \times 135.79$$

$$\boxed{\therefore P \Rightarrow 407.393\ kN}$$

Determine the crippling load of I-section joist 90cm x 40cm x 1cm of length 5m. One end of colum is hinged other end is fixed. Take $E = 2\times10^5\ N/mm^2$



I - 90cm x 40cm x 1cm

$L = 5m \Rightarrow 5000mm$

$l_e = \frac{L}{\sqrt{2}} \Rightarrow 0.70 \times 5 \Rightarrow 3500mm$

$E = 2\times10^5\ N/mm^2$

$$I_{xx} = \frac{BD^3}{12} - \frac{b_1 d_1^3}{12} \Rightarrow \frac{20\times40^3}{12} - \frac{19\times38^3}{12}$$

$$\Rightarrow 19786\times10^4\ mm^4$$

$$I_{yy} \Rightarrow \frac{20^3\times1}{12}\times2 + \frac{38\times1^3}{12} \Rightarrow 1336.5\times10^4\ mm^4$$

$$P = \frac{\pi^2 EI}{l_e^2} \Rightarrow \frac{\pi^2\times2\times10^5\times1336.5\times10^4}{3500^2} \Rightarrow 2.15\times10^6\ N$$

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12/03/2020

Rankine's - Gordon formula:-

Derivation:-

$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E}$$

P_R = Rankine's crippling load for Rankine's formula

P_C = crippling load $= \sigma_C \cdot A$

P_E = Euler's load $= \frac{\pi^2 EI}{le^2}$

$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E}$$

$$\frac{1}{P_R} \Rightarrow \frac{P_E + P_C}{P_C \cdot P_E}$$

$$P_R = \frac{P_C \cdot P_E}{P_E + P_C}$$

multiply & Diveded by P_E on right sides.

$$P_R = \frac{\frac{P_C \cdot \cancel{P_E}}{\cancel{P_E}}}{\frac{P_E + P_C}{P_E}}$$

$$P_R = \frac{P_C}{\frac{P_C}{P_E} + 1}$$

$$P_R = \frac{\sigma_C \cdot A}{\frac{\sigma_C \cdot A}{\frac{\pi^2 EI}{le^2}} + 1}$$

Radius of gyration $(K) = \sqrt{\frac{I}{A}}$

$[\because I = K^2 \times A]$

$$P_R = \frac{P_C}{\frac{\sigma_C \cdot \cancel{A}}{\pi^2 E} \cdot \frac{le^2}{(K^2 \times \cancel{A})} + 1}$$

Slenderness ratio $\lambda = \frac{l_e}{k}$

$$P_R = \frac{P_c}{1 + \frac{\sigma_c}{\pi^2 E} \cdot \lambda^2}$$

$$\alpha = \frac{\sigma_c}{\pi^2 E} = \text{Rankine's constant}$$

$$\boxed{\therefore P_R = \frac{P_c}{1 + \alpha\lambda^2}}$$

Sl. No	Material	σ_c in N/mm^2	'α'
1.	Wrought Iron.	250	$\frac{1}{9000}$
2.	Cast Iron	550	$\frac{1}{1600}$
3.	mild steel	320	$\frac{1}{7500}$
4.	Welded	50	$\frac{1}{750}$

1. The ex. & In. dia of hollow cast Iron Column 5cm & 4cm respectively, having length of 3m. which is both ends of the column are fixed. Take Rankine's constant $\alpha = \frac{1}{1600}$, $\sigma_c = 550\ N/mm^2$

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Given data.

Ex. dia = 5cm ⇒ 50mm

In. dia = 4cm ⇒ 40mm

length = 3m ⇒ 3000mm

$$\alpha = \frac{1}{1600}$$

$$\sigma_c = 550\ N/mm^2$$

$$P_R = \frac{P_c}{1+\alpha\lambda^2} \Rightarrow \frac{\sigma_c \cdot A}{1+\alpha\lambda^2}$$

$$\Rightarrow \frac{550 \times \frac{\pi}{4}(50^2-40^2)}{1+\left(\frac{1}{1600}\right)\left[\frac{le}{k}\right]^2}$$

$$\Rightarrow \frac{550 \times \frac{\pi}{4}(50^2-40^2)}{1+\left[\frac{1}{1600}\right]\left[\frac{3000/2}{16}\right]^2}$$

$$k = \sqrt{\frac{I}{A}}$$

$$k = \sqrt{\frac{\frac{\pi}{64}(50^4-40^4)}{\frac{\pi}{4}(50^2-40^2)}}$$

$$\Rightarrow 59.874 \times 10^3 N$$

$$\boxed{\therefore k = 16\ mm}$$

$$\boxed{\therefore P_R = 59.87\ KN}$$

2. 1.5m circular cast Iron column of dia. 5cm. one end of column is fixed another end is free. Take $\sigma_c = 560 N/mm^2$ Take $\alpha = \frac{1}{1600}$. Determine (1) Rankine's crippling load with F.O.S "3".

(2) Euler's crippling load with F.O.S "3"

$E = 1.7 \times 10^5\ N/mm^2$

$d_1 = 5cm \Rightarrow 5\times10 \Rightarrow 50mm$

$a = \frac{\pi}{4}(50)^2$

$l = 1.5m \Rightarrow 1500mm$

$\alpha = \frac{1}{1600}, \quad \sigma_c = 560 N/mm^2$

(i)

$$P_R = \frac{P_c}{1+\alpha\lambda^2} \Rightarrow \frac{P_c}{1+\alpha\left[\frac{l_e}{K}\right]^2}$$

$l_e = 2l$

$$K = \sqrt{\frac{I}{A}} \Rightarrow \sqrt{\frac{\frac{\pi}{64}(50)^4}{\frac{\pi}{4}(50)^2}} \Rightarrow 12.5mm$$

$$P_R \Rightarrow \frac{\sigma_c \times A}{1+\alpha\left(\frac{2l}{K}\right)^2} \Rightarrow \frac{560\times\frac{\pi}{4}(50)^2}{1+\frac{1}{1600}\left[\frac{2\times1500}{12.5}\right]^2}$$

$P_R \Rightarrow 29717.76N$

$$F.O.S = \frac{\text{Ultimate load}}{\text{working load}}$$

Ultimate load = 89153.305 N

$\boxed{\therefore \text{Ul. load} \Rightarrow 89.153KN}$

(ii)

$$P_E = \frac{\pi^2 EI}{l_e^2} \Rightarrow \frac{\pi^2\times\frac{\pi}{64}(50)^4\times1.2\times10^5}{(2\times1500)^2}$$

$\Rightarrow \cancel{57.89}KN$

$F.O.S = \frac{U.l}{W.l}$ $\quad$ $Ul = 3\times\cancel{57}.89$

$\boxed{\therefore \text{Ul. load} = \cancel{68.63}KN \; 171.584KN}$

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5. Determine the min. dia of circular cast Iron long column of length 4m. Both ends of the column are hinged. Take $\alpha = \frac{1}{1600}$ & $\sigma_c = 550 N/mm^2$ The ratio of internal dia to external dia is 0.8

length = 4m ⇒ 4000 mm   P = 250 KN

$$l_e = l = 4000 \text{ mm}$$

$$\alpha = \frac{1}{1600}$$

$$\sigma_c = 550 N/mm^2$$

$$\frac{d}{D} = 0.8$$

$$d = 0.8D.$$

$$P_R = \frac{P_c}{1+\alpha\lambda^2}$$

$$K = \sqrt{\frac{I}{A}}$$

$$K = \sqrt{\frac{\frac{\pi}{64}(D^4-(0.8D)^4)}{\frac{\pi}{4}[D^2-(0.8D)^2]}}$$

$$K = 0.32D$$

$$P_R = \frac{\sigma_c \times A}{1+\frac{1}{1600}\left[\frac{4000}{0.32D}\right]^2}$$

$$P_R = \frac{550 \times \frac{\pi}{4}(D^2-(0.8D)^2)}{1+\frac{1}{1600}\left[\frac{156.25 \times 10^6}{D^2}\right]}$$

$$P_R = \frac{155.5D^2}{1+\frac{97656.25}{D^2}}$$

$$250 \times 10^3 = \frac{155.5D^2}{\frac{D^2+97656.25}{D^2}}$$

$$(D^2+97656.25)250 \times 10^3 = 155.5D^4$$

$$250 \times 10^3 D^2 + 2.44 \times 10^4 = 155.5D^4$$

$$D^2[155.5D^2 - 250 \times 10^3] = 2.44 \times 10^4$$

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$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$155.5 D^4 - 250 \times 10^3 D^2 - 2.44 \times 10^4 = 0$$

$\therefore D = 115.58\,mm$

$\therefore d = 92.464\,mm$

4. Find the Euler's crushing load for a cylindrical cast Iron column 20cm external dia. and 25mm thick If it is 6m long and is hinged at both ends. Take $E = 2.1 \times 10^5\,N/mm^2$ compare the with the crushing load as given by the Rankine's formula. Taking $\sigma_c = 550\,N/mm^2$ & $\alpha = \frac{1}{1600}$ For what length of the column would these two formulae give the same crushing load.

Ex- dia(D) = 20cm ⇒ 200mm

$t = 25mm$ $\qquad d = D - 2t$

In- dia $= 200 - 2(25)$

⇒ 150mm

$l_e = l = 6m \Rightarrow 6000mm$

$E = 2.1 \times 10^5\,N/mm^2$

$\sigma_c = 550\,N/mm^2$

$\alpha = \frac{1}{1600}$

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$$P_R = P_C + P_E$$

$$P_R = \frac{P_C}{1+\alpha\lambda^2} \Rightarrow \frac{\sigma_C \cdot A}{1+\alpha\left[\frac{l_e}{k}\right]^2}$$

$$k = \sqrt{\frac{I}{A}}$$

$$k = \sqrt{\frac{\frac{\pi}{64}(200^4-150^4)}{\frac{\pi}{4}(200^2-150^2)}}$$

$$k = 62.5\ mm$$

$$\Rightarrow \frac{550 \times \frac{\pi}{4}(200^2-150^2)}{1+\frac{1}{1600}\left[\frac{6000}{62.5}\right]^2}$$

$$\Rightarrow 1.118 \times 10^6\ N$$

$$P_R \Rightarrow 1.118 \times 10^3\ kN$$

$$P_R = P_E$$

$$\frac{\sigma_C \cdot A}{1+\alpha\left[\frac{l_e}{k}\right]^2} = \frac{\pi^2 EI}{l_e^2}$$

$$\frac{550 \times \frac{\pi}{4}(200^2-150^2)}{1+\frac{1}{1600}\left[\frac{6000}{62.5}\right]^2} = \frac{\pi^2 \times 2.1 \times 10^5 \times \frac{\pi}{64}(200^4-150^4)}{l_e^2}$$

$$\frac{7.559 \times 10^6}{1+1.6\times10^{-7} l_e^2} = \frac{1.41 \times 10^{14}}{l_e^2}$$

$$1.41 \times 10^{14} + 27.80 \times 10^6 l_e^2 = l_e^2 \times 7.559 \times 10^6$$

$$-13.044 \times 10^6 l_e^2 = 1.41 \times 10^{14}$$

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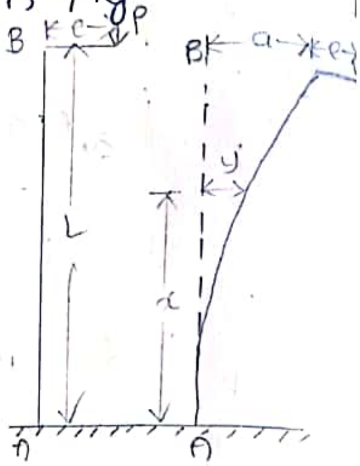
30/03/2020

Column with Eccentric load :-

Shows a column AB of length 'l' fixed at end "A" and free at end "B". The column is subjected to a load 'P' which is eccentric by amount of 'e'. The free end will side ways by a mount of 'a' and the column will deflect as shown in fig.



a = deflection at free end "B".

e = Eccentricity

A = Area of C/S of column

consider any section at a distance "x" from fixed end "A".

let y = deflection at the section then

moment at the section $= P(a+e-y)$

But moment is also $= EI\dfrac{d^2y}{dx^2}$

$$EI\frac{d^2y}{dx^2} = P(a+e-y)$$
$$= P(a+e) - Py$$

$$EI\frac{d^2y}{dx^2} + Py = P(a+e)$$

Divide both sides with "EI"

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = \frac{P}{EI}(a+e)$$

Where $k^2 = \dfrac{P}{EI}$, $k = \sqrt{\dfrac{P}{EI}}$

$$\frac{d^2y}{dx^2} + k^2 y = k^2(a+e)$$

The complete solution of above equation

$$y = C_1\cos(kx) + C_2\sin(kx) + (a+e) \rightarrow (1)$$

Boundary conditions:-

(i) slope $\dfrac{dy}{dx} = 0$, $x = 0$

Differentiation eq(1) w.r.to "x".

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$$\frac{dy}{dx} = -c_1 \sin(kx)\,k + c_2 \cos(kx)\cdot k + 0 \longrightarrow (2)$$

$0 = 0 + c_2 k$ $\quad [\because k = \sqrt{P/EI}$ cannot be zero$)$

$\boxed{c_2 = 0}$

(ii) Deflection $x = 0,\ y = 0$

$$0 = c_1 + (a+e)$$

$$c_1 = -(a+e)$$

substitute c_1, c_2 values in eq(1)

$$y = -(a+e)\cos(kx) + 0 + (a+e)$$

$$y = -(a+e)\cos(kx) + (a+e) \longrightarrow (3)$$

at $x = l$, $y = a$, hence eq(3) became

$$a = -(a+e)\cos(kl) + (a+e)$$

$$(a+e)\cos(kl) = \cancel{a} + e - \cancel{a}$$

$$a+e = \frac{e}{\cos(kl)}$$

$$(a+e) = e\sec(kl) \longrightarrow (4).$$

Maximum stress:-

Let us find the max compressive stress for the column section. Due to ecentricity there will be bending stress and also direct stress.

$$\sigma_{max} = \sigma_a + \sigma_b$$

where σ_a = direct stress $\Rightarrow P/A$

σ_b = max. bending stress

The max. bending stress will be at the section where bending moment is max. B.M. max at free end.

$$\text{Max. B.M} = P(a+e)$$

$$M = Pe(\sec(kl)) \quad [\because \text{from eq(4)}]$$

using $\dfrac{M}{I} = \dfrac{\sigma_b}{y}$

$$\sigma_b = \frac{M}{I} \times y$$

$$\sigma_b = \frac{M}{Z}$$

where $Z = I/y$ section modulus

$$\sigma_b = \frac{P \times e \sec(Kl)}{Z} \qquad [\because M = Pe \sec(Kl)]$$

Hence the max comp. stress become as

$$\sigma_{max} = \frac{P}{A} + \frac{P \times e \sec(Kl)}{Z}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Pe \sec(l \times \sqrt{P/EI})}{Z}$$

$[\because \text{where } k = \sqrt{\frac{P}{EI}}$

The eqn is used for a column whose one end is fixed, other end is free and the load is eccentric to the column. The relation b/w the actual length and effective length for a column whose one end is fixed and other end is free is given by

$$l_e = 2l \Rightarrow l = \frac{l_e}{2}$$

$$\sigma_{max} = \frac{P}{A} + \frac{P \times e \sec\left[\frac{l_e}{2} \times \sqrt{\frac{P}{EI}}\right]}{Z}$$

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Problems on Eccentric loaded column

① A column of circular section is subjected to a load of 120KN. The load is parallel to the axis but eccentric by an amount of 2.5mm. The external and internal diameter of columns are 60mm & 50mm respectively. If both the ends of the columns are hinged and column is 2.1m long, then determine the maximum stress in the column. Take E = 200 → GN/m^2.

GN/m^2

Soln.

$$\sigma_{max} = P/A + \frac{P \times e \times \sec\left(\frac{L_e}{2} \times \sqrt{P/EI}\right)}{Z}$$

Given data

Load $P = 120KN = 120 \times 10^3 N$

Eccentricity, $e = 2.5m = 2.5 \times 10^{-3} m$

where A = area of c/s

$$= \pi/4 (D^2 - d^2) = \pi/4 (0.06^2 - 0.05^2)$$

$$= \pi/4 \times 0.0011 = 8.639 \times 10^{-4} m^2$$

$$I = \text{moment of Inertia} = \frac{\pi (D^4 - d^4)}{64}$$

$$Z = I/y \quad \therefore y = D/2$$

$$Z = \frac{\pi/64 (D^4 - d^4)}{D/2} = \frac{\pi/64 (0.06^4 - 0.05^4)}{0.03}$$

$$= \frac{\pi (1.296 \times 10^{-5} - 0.625 \times 10^{-5})}{64 \times 0.03}$$

$$\boxed{Z = 1.0975 \times 10^{-5} m^3}$$

$$\sec\left(\frac{L_e}{2}\times\sqrt{P/EI}\right) = \sec\left(\frac{2.1}{2}\times\sqrt{\frac{120\times10^3}{200\times10^9\times0.329\times10^{-5}}}\right)$$

$$= \sec\,(1.4179 \text{ radians})$$

$$= \sec\,(81.239) = \underline{6.566}$$

$$\because \left(1.4179\times 180/\pi = 81.239\right)$$

Substitute all values in eqn (1) :-

$$\sigma_{max} = \frac{120\times10^3}{8.639\times10^{-4}} + \frac{(120\times10^3)\times(2.5\times10^{-3})\times6.566}{1.0975\times10^{-5}}$$

$$\boxed{\sigma_{max} = 318.38\times10^6 \text{ N/m}^2} \text{ (or) } \boxed{318.38 \text{ N/mm}^2}$$

(2) If the given column of 19.22 is subjected to an eccentric load of 100KN and maximum Permissible stress is limited to $320\ MN/m^2$, then determine the maximum eccentricity of the load.

Soln

$D = 60mm$, $d = 50mm$

$= 0.06m$, $d = 0.05m$

$l = 2.1m$, $L_e = l = \underline{2.1m}$

$E = 200\ GN/m^2 = 200\times10^3\ N/m^2$, $I = 0.0329\times10^{-5}\ m^4$

$Z = 1.0975\times10^{-5}\ m^3$, $A = 8.639\times10^{-4}\ m^2$

Eccentric load, $P = 100KN = 100\times10^3\ N$

$\sigma_{max} = 320\times10^6\ N/m^2$

Let e = maximum eccentricity

$$\sigma_{max} = P/A + \frac{P \times e \times \sec\left(\frac{l_e}{2} \times \sqrt{P/EI}\right)}{Z}$$

let us find the value of $\sec\left(\frac{l_e}{2} \times \sqrt{P/EI}\right)$.

$$\sec\left(\frac{l_e}{2} \times \sqrt{P/EI}\right) = \sec\left[\frac{2.1}{2} \times \sqrt{\frac{100 \times 10^3}{200 \times 10^9 \times 0.0329 \times 10^{-5}}}\right]$$

$$= \sec(1.294 \text{ rads}) \quad \left[1.294 \times \frac{180^\circ}{\pi} = \right]$$

$$= \sec(74.16^\circ) = 3.665$$

Substitute these value in eqn ①.

$$320 \times 10^6 = \frac{100 \times 10^3}{8.639 \times 10^{-4}} + \frac{(100 \times 10^3) \times e \times 3.665}{1.0975 \times 10^{-5}}$$

$$320 \times 10^6 = 115.754 \times 10^6 + 33394e \times 10^6$$

$$320 = 115.754 + 33394e$$

$$\boxed{e = 6.116 mm}$$

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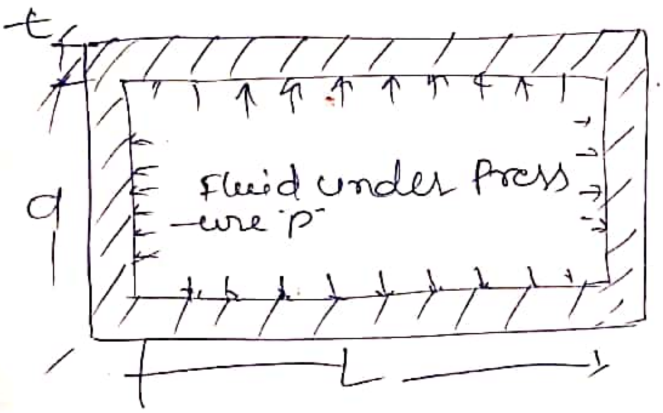
Thin cylinders and spheres

Introduction :-

The vessels such as boilers, compressed air receivers etc., are of cylindrical and spherical forms. These vessels are generally used for storing fluids (liquid or gas) under pressure. The walls of such vessels are thin as compared to their diameters. If the thickness of the wall of the cylindrical vessel is less than 1/15 to 1/20 of its internal diameter, the cylindrical vessel is known as a thin cylinder. In case of thin cylinder, the stress distribution is assumed uniform over the thickness of the wall.

Thin cylindrical vessel subjected to internal pressure :-



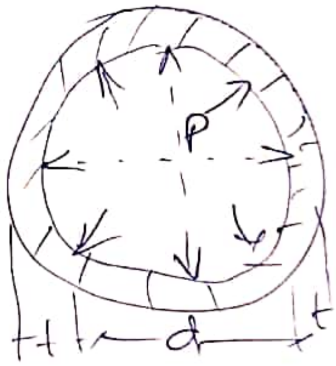


Fig shows a thin cylindrical vessel in which a fluid under pressure is stored.

Let d = Internal diameter of the thin cylinder

t = thickness of the wall of the cylinder

p = Internal pressure of the fluid

L = Length of the cylinder.

On account of the Internal Pressure P, the Cylindrical vessel may fail by splitting up in any one of the two ways as shown in fig

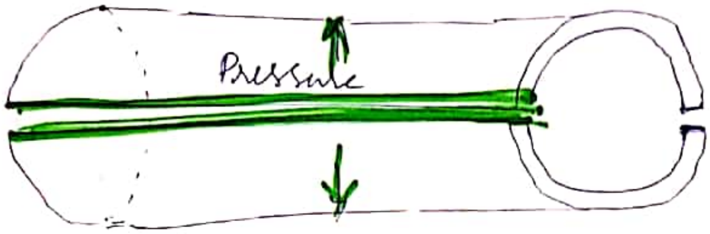
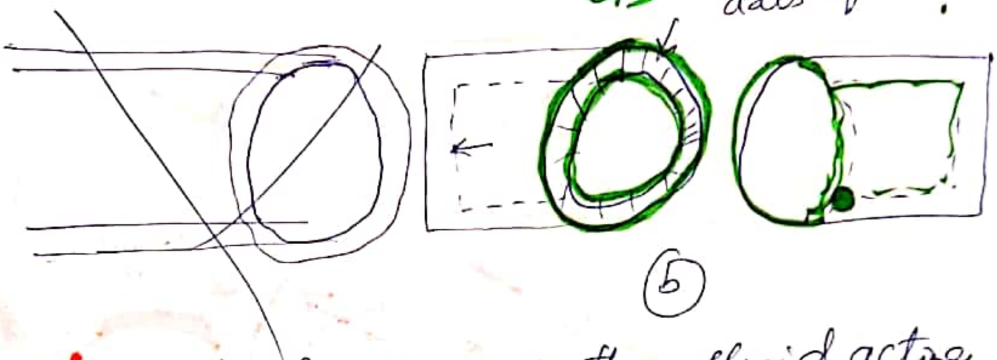


fig (a)



(b)

The forces, due to Pressure of the fluid acting vertically upwards and downwads on the thin cylinder, tend to burst the cylinder (a)

The force due to Pressure of the fluid, acting at the ends of the thin cylinder, tend to burst the thin cylinder (b).

Stress in a thin cylinder vessel Subjected to Internal Pressure :-

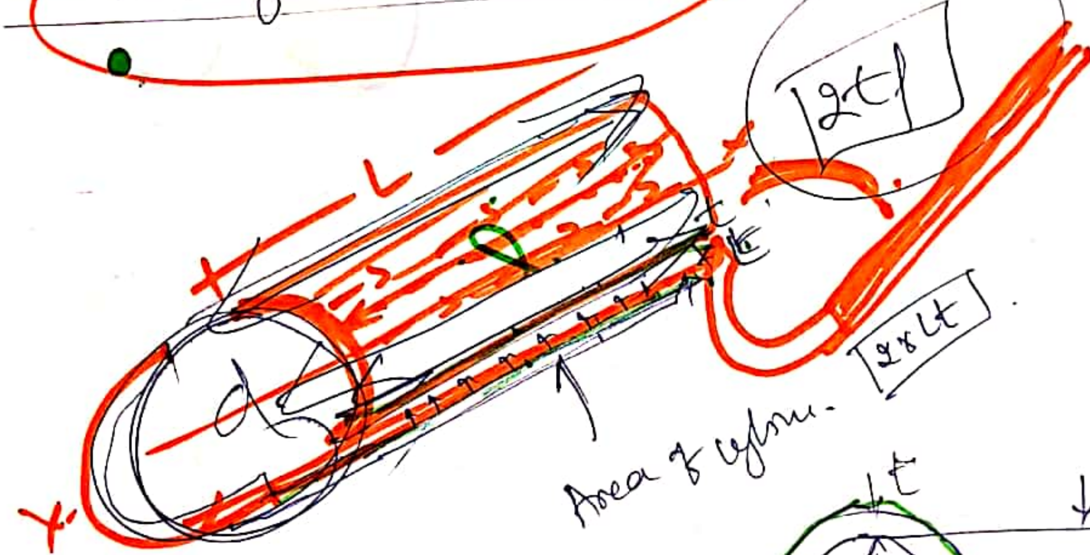
When the thin cylindrical vessel is subjected to Internal fluid Pressure, the stress in the wall of the cylinder on the c/s along the axis and on the c/s perpendicular to the axis are set up. These stresses are tensile & are known as:

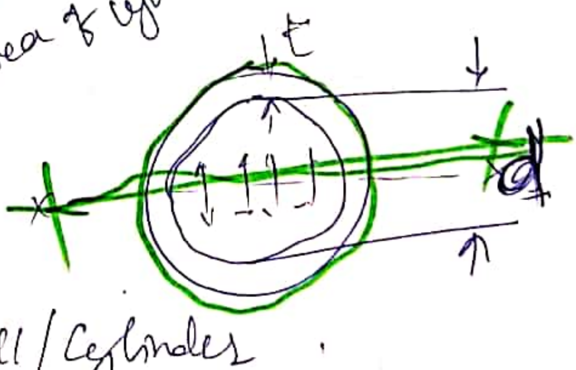
1. circumferential stress (or hoop stress) &

2. longitudinal stress.

The name of the stress is given according to the direction in which the stress is acting. The stress is acting along the circumferences of the cylinder is called circumferential stress where as the stress acting along the length of the cylinder (i.e, in the longitudinal direction) is known as longitudinal stress. The circumferential stress is also known as hoop stress.

* circumferential stress :





let l = length of the cylinder

d = diameter of the shell / cylinder.

t = thickness of cylinder

p = pressure intensity

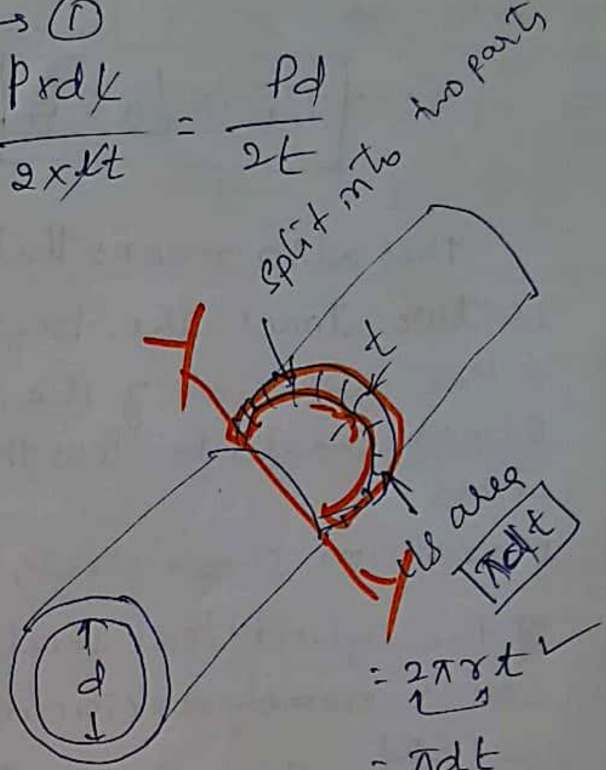
Total force developed by the system

$= \text{stress} \times \text{Area}$

$= p \times d L \rightarrow (1)$

$\sigma_H = P/A = \dfrac{p \times d \cancel{L}}{2 \times \cancel{L} t} = \dfrac{Pd}{2t}$

$$\boxed{\sigma_c = \sigma_H = \frac{Pd}{2t}}$$



$= 2\pi r t$

$= \pi d t$

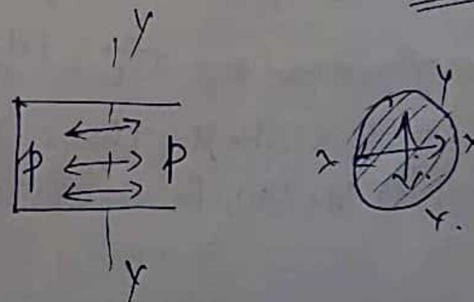
longitudinal stress :-

let $= p$ = Pressure intensity

l = length of cylinder

d = diameter of the cylinder

t = thickness of cylinder



force due to Pressure Intensity

$= p \times \pi d^2/4$

$\therefore \sigma_L = \dfrac{\text{force}}{\text{c/s area}}$

$= \dfrac{p \times \pi d^2/4}{2\pi r t} = \dfrac{p \times \cancel{\pi} d^{\cancel{2}}/4}{2 \cancel{\pi} \cancel{r} t} = \dfrac{Pd}{4t}$

$$\boxed{\sigma_L = Pd/4t}$$

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$$\boxed{\sigma_c = \frac{Pd}{2t}} \qquad \boxed{\sigma_l = \frac{Pd}{4t}}$$

$$\therefore \boxed{\sigma_L = \tfrac{1}{2}\sigma_c}$$

This also means that circumferential stress (σ_c) is two times the longitudinal stress (σ_L) Hence in the material of the cylinder the permissible stress should be less than than the circumferential stress.

Maximum shear stress : At any point in the material of the cylindrical shell, there are two principal stress, namely a circumferential stress of magnitude $\sigma_1 = \frac{Pd}{2t}$ acting circumferentially & a longitudinal stress of magnitude $\sigma_L = Pd/4$ acting parallel to the axis of the shell. These two stresses are tensile & perpendicular to each other.

$\therefore$ maximum shear stress $\tau_{max} = \frac{\sigma_c - \sigma_l}{2}$

$$= \frac{Pd/2t - Pd/4t}{2} = \frac{4Pd - 2Pd}{8\times 2t} = \frac{2Pd}{16t} = \frac{Pd}{8t}$$

Problems :- A cylindrical pipe of diameter of 1.5m and thickness 1.5cm is subjected to an internal fluid pressure of 1.2 N/mm². Determine;

① longitudinal stress developed in the pipe, and

(2) circumferential stress developed in the pipe.

Sol^n $d = 1.5m$

$t = 1.5cm = 1.5\times10^{-2} m$

$p = 1.2 N/mm^2$

As the ratio $= \frac{1.5\times10^{-2}}{1.5} = \frac{1}{100}$ which is less than $1/20$, hence this is a thin cylinder

(1) longitudinal stress $\sigma_2 = \frac{pd}{4t}$

$= \frac{1.2\times1.5\times10^{2}}{4\times1.5\times10^{-2}} = 30 N/m^2$

(ii) The circumferential stress (σ_c) is given by

$\sigma_c = pd/2t = \frac{1.2\times1.5}{2\times1.5\times10^{-2}} = 60 N/mm^2$

② A cylinder of internal diameter 2.5m and of thickness 5cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm², determine the internal pressure of the gas.

Sol^n Internal dia of cylinder, $d = 2.5m$

Thickness of cylinder, $t = 5cm = 5\times10^{-2} m$

maximum permissible stress $= 80 N/mm^2$

Let P = Internal Pressure of the gas

$$\sigma_1 = \frac{Pd}{2t}$$

$$P = \frac{2t \times \sigma_1}{d} = \frac{2 \times 5 \times 10^{-2} \times 80}{2.5} = 3.2\ N/mm^2$$

③ A thin cylinder of Internal diameter 1.25m contains a fluid at an internal Pressure of $2N/mm^2$. Determine the maximum thickness of the cylinder if.

(i) The Longitudinal Stress is not to exceed $30N/mm^2$

(ii) $\sigma_c = 45N/mm^2$

given data $d = 1.25m$, $P = 2N/mm^2$, $\sigma_c = 45N/mm^2$, $\sigma_L = 30\ N/mm^2$

$\sigma_c = \frac{Pd}{2t}$ $t = 0.0277m$ $t = \frac{Pd}{\sigma_c t}$

$\sigma_L = Pd/4t$ $t = 0.0208m$

Effeciency of a Joint

The cylinderical shells such as boilers are having two types of Joints namely longitudinal Joint and circumferential Joint. In case of Joint holes are made in the material of the shell for the rivets. Due to the holes, the area offering resistance decreasing. Due to decreasing in area the stress (which is the equal to the force divided by the area) developed in the material of the shell will be more.

Hence in case of rivetted shell be circumferential and longitudinal stresses are greater than what are given by eqns in σ_c & σ_L, if the efficiency of a longitudinal joint & circumferential joint are given then the circumferential & longitudinal stresses are.

Let η_L = efficiency of a longitudinal joint

η_c = efficiency of the circumferential joint

then $$\boxed{\sigma_c = \frac{Pd}{2t \times \eta_L}}$$

and $$\boxed{\sigma_L = \frac{Pd}{4t \times \eta_c}}$$

<u>Note</u>:- (i) In longitudinal joint, the circumferential stresses is developed in circumferential joint, the longitudinal stress is developed.

(ii) efficiency of a joint means the efficiency of a longitudinal joint.

(iii) Thickness of thin cylinder can be determine by given above formula.

① A boiler is subjected to an internal steam pressure of $2 N/mm^2$. The thickness of boiler plate is 2.0 cm and permissible tensile stress is $120 N/mm^2$. Find out the maximum diameter, when efficiency of longitudinal joint is 40% and that of circumferential joint is 90%.

$\therefore \eta_c = 90\% = 0.9$

$\eta_L = 40\% = 0.4$

(i) $\sigma_c = \frac{P \times d}{2 \times \eta_{l} \times t} = \frac{2 \times d}{2 \times 0.9 \times 2}$

$120 = \frac{2 \times d}{3.6} = \underline{216\,cm}$

(ii) $\sigma_t = 120 = \frac{2 \times d}{4 \times 0.4 \times 2} = \underline{192\,cm}$

(σ_c)

The longitudinal @ circumferential stresses induced in the material ~~are directly perpendi~~ ~~cular to the~~ are directly proportional to diameter (d). Hence the stress induce is less if value of d is less, Hence take the minimum value of above two.

② A boiler shell is to be made of 15mm thick plate having a limiting tensile stress of 120 N/mm². If the efficiency of the longitudinal and circumferential joints are 70% and 30% respectively determine

(i) The maximum permissible diameter of the shell for an internal pressure of 2 N/mm²,

(ii) Permissible intensity of internal pressure when the shell diameter is 1.5m.

Soln

given data

$t = 15\,mm$

tensile stress = $120\,N/mm^2$

$\eta_l = 70\%$

$\eta_c = 30\%$

$\eta_l = 70\% = 0.70$

$\eta_c = 30\% = 0.30$

$\therefore P = 2 N/mm^2$

(a) $\sigma_c = \frac{Pd}{2t\eta_l}$

$$120 = \frac{2 \times d}{2 \times 0.7 \times 15}$$

$$\boxed{d = 1260 mm}$$

(b) Taking limiting tensile stress = Longitudinal stress

$(\sigma_2) = 120 N/mm^2$

$\sigma_2 = 120 N/mm^2$

$$\sigma_l = \frac{Pd}{4 \times \eta_c \times t}$$

$$120 = \frac{2 \times d}{4 \times 0.3 \times 15}$$

$$\boxed{d = 1080 mm}$$

$\therefore \boxed{d = 1080 mm}$

(ii) Permissible Intensity of Internal Pressure when the shell diameter 1.5m

$d = 1.5m = 1500 mm$

(a) Taking limiting tensile stress = circumferential stresses $(\sigma_c) = 120 N/mm^2$

$\sigma_c = \frac{P \times d}{2\eta_l \times t} \quad 120 = \frac{P \times 1500}{2 \times 0.7 \times 15}$

$\boxed{P = 1.44 N/mm^2}$

$\underline{P = 1.68 \, N/mm^2}$

(b) Taking limiting tensile stress = Longitudinal stress (σ_L)

$= 120 \, N/mm^2$.

$$\sigma_L = \frac{Pd}{4 \times \eta_c \times t}$$

$$120 = \frac{P \times 1500}{4 \times 0.3 \times 15}$$

$$\therefore \boxed{P = 1.44 \, N/mm^2}$$

— Hence in order both the conditions may be satisfied the maximum permissible internal pressure is equal to the minimum value of give (i) & (ii)

$$\sigma_L = \frac{P \times d}{4 \times \eta_c \times t} = \frac{1.68 \times 1500}{4 \times 0.30 \times 15} = 140 \, N/mm^2.$$

The value is more than limiting value

Effect of Internal Pressure on the dimensions of a Thin cylindrical shell.

When a fluid having internal pressure (P) is stored in a thin cylindrical shell, due to internal pressure of the fluid the stresses set up at any point of the material of the shell are.

(i) Hoop or circumferential stress (σ_c), acting on longitudinal section.

(ii) Longitudinal stress (σ_L) acting on the circumferential section.

These stresses are Principal stresses, as they are acting on principal planes is zero as the thickness (t) of the cylinder is very small. Actually the stresses in the third principal plane is radial stress which is very small for the thin cylinders & can be neglected.

Let P = Internal Pressure of fluid.

L = length of cylindrical,

d = diameter of the cylindrical shell.

t = thickness of cylindrical sheel.

E = Modulus of Elasticity for the material of the shell.

σ_c = Hoop stress in the material

σ_L = longitudinal stress in the material

μ = Poisson's ratio

δd = change in diameter due to stresses set in the material

δL = change in length

δV = change in volume

The value of σ_1 & σ_2 are given by equations

$$\sigma_c = \frac{pd}{2t}$$

$$\sigma_L = pd/4t$$

let e_c = circumferential strain

e_L = Longitudinal strain

Then circumferential strain,

$$e_c = \frac{\sigma_c}{E} - \frac{\mu \sigma_L}{E}$$

$$e_c = \frac{pd}{2tE} - \frac{\mu pd}{4tE}$$

$$\boxed{e_c = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right)}$$

and longitudinal strain,

$$e_L = \frac{\sigma_L}{E} - \frac{\mu \sigma_c}{E}$$

$$= \frac{pd}{4tE} - \frac{\mu pd}{2tE}$$

$$\boxed{e_L = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)}$$

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But circumferential strain is also given as,

$$e_c = \frac{\text{change in circumference due to Pressure}}{\text{original circumference}}$$

$$= \frac{\text{Final circumference - original circumference}}{\text{original circumference}}$$

$$= \frac{\pi(d+\delta d) - \pi d}{\pi d}$$

$$= \frac{\cancel{\pi d} + \pi \delta d - \cancel{\pi d}}{\pi d} = \frac{\cancel{\pi}\delta d}{\cancel{\pi} d} = \frac{\delta d}{d}$$

or $\frac{\text{change in } d}{\text{original } d}$

equating two value of e_c

$$\frac{\delta d}{d} = \frac{Pd}{2tE}\left(1 - \mu/2\right)$$

$$\boxed{\delta d = \frac{Pd^2}{2tE}\left(1 - \mu/2\right)}$$

Similarly, longitudinal strain is also given as

$$e_L = \frac{\text{change in length}}{\text{original length}} = \frac{\delta L}{L}$$

Equating two value of e_L

$$\frac{\delta L}{L} = \frac{Pd}{2tE}\left(1/2 - \mu\right)$$

$$\boxed{\delta L = \frac{PdL}{2tE}\left(1/2 - \mu\right)}$$

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But change in volume (δv) = Final volume - original - Value.

original volume of v = Area of cylindrical shell × length

$$= \frac{\pi}{4} d^2 \times L$$

Final volume = Final Area of circum × Final length

$$= \frac{\pi}{4} (d+\delta d)^2 \times (L+\delta L)$$

$$= \frac{\pi}{4}\left(d^2 + (\delta d)^2 + 2d\delta d\right) \times (L+\delta L)$$

$$= \frac{\pi}{4}\left(d^2L + (\delta d)^2 L + 2d\delta d L + \delta L d^2 + (\delta d)^2(\delta L) + 2d(\delta d)(\delta L)\right)$$

Neglecting the smaller quantities such as $(\delta d^2)L$, $\delta L(\delta d)^2$ and $2d(\delta d)(\delta L)$

Final volume $= \frac{\pi}{4}\left(d^2 + 2d\delta d L + \delta L d^2\right)$

∴ change in volume (δv)

$$= \frac{\pi}{4}\left(d^2L + 2d\delta d L + \delta L d^2\right) - \frac{\pi}{4} d^2 \times L$$

$$= \frac{\pi}{4}\left[2d\delta d L + \delta L d^2\right)$$

∴ volumetric strain $= \frac{\delta v}{v} = \frac{\cancel{\pi/4}\left(2d\delta d L + \delta L d^2\right)}{\cancel{\pi/4}\, d^2 \times L}$

$$= \frac{\cancel{2\delta d}\ \frac{\pi}{4} 2d\delta d L + \frac{\pi}{4}\delta L d^2}{\frac{\pi}{4} d^2 \times L}$$

$$= \frac{2d\delta d L + \delta L d^2}{d^2 \times L}$$

$$= \frac{2d\delta d \cancel{L}}{d^{\cancel{2}} \cancel{L}} + \frac{\delta L \cancel{d^2}}{\cancel{d^2} \times L}$$

$$= 2\delta d/d + \delta L/L$$

$$= \frac{2\delta d}{d} + \frac{\delta L}{L}$$

$$\therefore e_c = \frac{\delta d}{d}$$

$$= 2e_c + e_L \qquad e_L = \frac{\delta L}{L}$$

$$= 2 \cdot \frac{Pd}{2Et}(1 - \mu/2) + \frac{Pd}{2Et}\left(\tfrac{1}{2} - \mu\right)$$

$$= \frac{Pd}{2Et}\left(2 - \frac{2\mu}{2} + \frac{1}{2} - \mu\right)$$

$$= \frac{Pd}{2Et}\left(\cancel{\frac{4-2\mu}{2}} + 2 - \mu + \frac{1}{2} - \mu\right)$$

$$\boxed{\text{Volumetric strain} = \frac{Pd}{2Et}\left(\tfrac{5}{2} - 2\mu\right)}$$

$$\boxed{\text{Also change in volume} = V(2e_c + e_L)}$$

(2) A cylindrical shell 90cm long 20cm internal diameter having thickness of metal as 8mm is filled with fluid at atmospheric pressure. If an additional $20\,cm^3$ of fluid is pumped into the cylinder, find (i) the pressure exerted by the fluid on the cylinder and (ii) the hoop stress induced. Take $E = 2\times10^5$ N/mm^2 & $\mu = 0.3$

Sol^n Length of cylinder, $L = 90\,cm$

$d = 20\,cm$

$t = 8\,mm = 0.8\,cm$

$\delta V = 20\,cm^3$

$V = \pi/4\, d^2 \times L = \pi/4 \times 20^2 \times 90 = 28274.33\ cm^3$

(i) Let P = Pressure exerted by fluid on the cylinder.

we know equation

volumetric strain > $\frac{\delta v}{V} = \frac{(2e_c + e_l)}{\cancel{2(e_c+e_l)}} \rightarrow (1)$

$$\frac{\delta v}{V} = 2\left(\frac{Pd}{2tE}\left(1-\frac{\mu}{2}\right) + \frac{Pd}{2tE}\left(\frac{1}{2}-\mu\right)\right)$$

$$\frac{20}{28274.33} = \frac{2pd}{2Et}\left[1-\frac{\mu}{2}+\frac{1}{2}-\mu\right)$$

$$\frac{20}{28274.33} = \frac{\cancel{2}pd}{\cancel{2}tE}\left(1-\frac{\mu}{2}\right) + \frac{Pd}{2tE}\left(\frac{1}{2}-\mu\right)$$

$$= \frac{P\times 20}{2\times10^5\times0.8}\left(1-\frac{0.3}{2}\right) + \frac{P\times 20}{0.8\times2\times10^5}\left(\frac{1}{2}-0.3\right)$$

$$0.000707 = \frac{P}{8000}\times0.85 + \frac{P}{8000}\times0.2 = \frac{1.05P}{8000}$$

$$\therefore P = \frac{8000\times0.000707}{1.05} = 5.386\ N/mm^2.$$

(ii) Hoop stress

$$\sigma_c = \frac{Pd}{2t} = \frac{5.386\times20}{2\times0.8} = 67.33\ N/mm^2$$

(*) A cylindrical thin drum 80cm in diameter and 3m long has a shell thickness of 1cm. If the drum is subjected to an internal pressure of $2.5\,N/mm^2$, determine (i) change in diameter (ii) change in length & (iii) change in volume.
Take $E = 2\times10^5\,N/mm^2$; poisson's ratio $\mu = 0.25$

Soln

Given data:-

$d = 80cm$

$L = 3m = 3\times100 = 300cm$

$t = 1cm$

$P = 2.5\,N/mm^2$

$E = 2\times10^5\,N/mm^2$

$\mu = 0.25$

(iii) using eqn volumetric strain.

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta L}{L}$$

$$\frac{\delta V}{V} = 2\times\frac{0.035}{80} + \frac{0.0357}{300}$$

$$\boxed{\delta V = 0.001\times V}$$

$$V = \frac{\pi}{4}d^2\times L = \frac{\pi}{4}\times 80^2\times 300$$
$$= 1507964.47\,cm^3$$

(i) change in diameter (δd) is given by equation as

$$\delta d = \frac{Pd^2}{2tE}\left(1 - \frac{1}{2}\times\mu\right)$$

$$\delta d = \frac{2.5\times(80)^2}{2\times1\times2\times10^5}\left(1 - \frac{1}{2}\times0.25\right)$$

$$= 0.04\,[1 - 0.125] = 0.035\,cm$$

(ii) change in length (δL) is given by

$$\delta L = \frac{PdL}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{2.5\times80\times300}{2\times1\times2\times10^5}\left[\frac{1}{2} - 0.25\right]$$

$$= 0.0357\,cm$$

② A cylindrical vessel whose ends are closed by means of rigid flange plates, is made of steel plate 3mm thick. The length of the internal diameter of the vessel are 50cm & 25cm respectively. Determine the longitudinal and hoop stresses in the cylindrical shell due to an internal fluid pressure of 3 N/mm². Also calculate the increase in length, diam diameter and volume of the vessel. Take $E = 2\times10^5 N/mm^2$ & $\mu = 0.3$

Soln :-

$L = 50cm$

$d = 25cm$

$p = 3 N/mm^2$

$t = 3mm = 0.3cm$

$E = 2\times10^5 N/mm^2$

$\mu = 0.3$

σ_c = Hoop stress, $= \frac{pd}{2t} = \frac{3\times25}{2\times0.3} = 125 N/mm^2$

$\sigma_L = \frac{pd}{4t} = \frac{3\times25}{4\times0.3} = 62.5 N/mm^2$

using equation for circumferential stress

its -

$$e_c = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

$$e_c = \frac{125}{2\times10^5} - \frac{0.3\times62.5}{2\times10^5} = 53.125\times10^{-5}$$

But circumferential stress is given by

$$e_c = \frac{\delta d}{d}$$

Equating two e_c values

$$\delta d = d \times 53.125 \times 10^{-5} = 25 \times 53.125 \times 10^{-5} = \underline{0.0133 cm}$$

$\therefore$ Increasing diameter $= \underline{0.0133 cm}$

$$\delta L = \frac{PdL}{2tE}\left(\frac{1}{2} - \frac{1}{m}\right)$$

$$\delta L = \frac{3 \times 25 \times 50}{2 \times 1.3 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right)$$

$$= \frac{3750}{120{,}000}(0.2)$$

$$= 0.03125 \times 0.2 = \underline{\underline{0.00625 cm}}$$

volumetric strain:-

$$\frac{\delta V}{V} = 2\frac{\delta d}{d} + \frac{\delta L}{L}$$

$$\frac{\delta V}{V} = 2 \times \frac{0.0133}{25} + \frac{0.00625}{50} =$$

$$\delta V / V = 0.001064 + 0.000125$$

$$= 0.001189$$

$$V = \frac{\pi d^2}{4} \times L = \frac{\pi (25)^2}{4} \times 50 = 24{,}546.875 \text{ cm}^3$$

$$\delta V = 0.001189 \times 24{,}546.875 =$$

$$\boxed{\delta V = 29.186 \text{ cm}^3}$$

* Initial Difference In Radii at the Junction of a Compound cylinder for Shrinkage.

Sol^n :- By shrinking the outer cylinder over the Inner cylinder, some compressive stresses are produced in the Inner cylinder. In order to shrink the outer cylinder over the Inner cylinder. The Inner diameter of the outer cylinder is heated and inner cylinder is inserted into it.. after the cooling the outer cylinder shrinks over the Inner cylinder. Thus Inner cylinder is put into compression & outer cylinder is put into tension. After. After shrinking the outer radius of Inner cylinder decreases whereas the Inner radius of outer cylinder Increases from the Initial values.

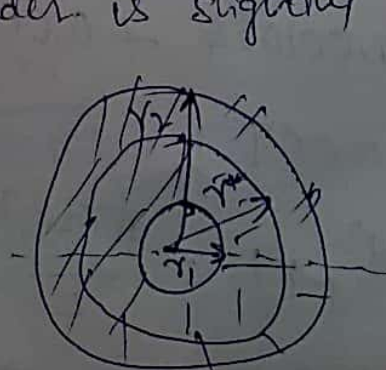
let r_2 = outer radius of the outer cylinder.

r_1 = Inner radius of Inner cylinder.

r^* = Radius of Junction after shrinking @ it is common radius after shrinking

p^* = Radial Pressure at the Junction after shrinking

Before shrinking the outer radius of the Inner cylinder is slightly more than r^* and inner radius of the outer cylinder is slightly less than r^*.



For the outer and Inner cylinder Lame's equations are used. these equations are

$$P_x = \frac{b}{x^2} - a \text{ and } \sigma_x = \frac{b}{x^2} + a$$

The value of constants a & b will be different for each cylinder.

Let the constants for inner cylinder be a_2, b_2 & for outer cylinder a_1, b_1.

The radial Pressure at the Junction (i.e, p^*) is same for outer cylinder & Inner cylinder.

At the Junction, $x = r^*$ & $P_x = p^*$. Hence radial pressure at the Junction.

$$p^* = \frac{b_1}{r^{*2}} - a_1 = \frac{b_2}{r^{*2}} - a_2 \rightarrow (A)$$

$$\frac{b_1 - b_2}{r^{*2}} = (a_1 - a_2)$$

$$(b_1 - b_2) = r^{*2}(a_1 - a_2) \longrightarrow (B)$$

Now the hoop strain (or circumferential strain) in the cylinder at any point.

$$= \frac{\sigma_x}{E} + \frac{P_x}{mE} \longrightarrow (C)$$

But circumferential strain

$$= \frac{\text{Increase in circumference}}{\text{original circumference}}$$

$$= \frac{2\pi(r + dr) - 2\pi r}{2\pi r} = \frac{\cancel{2\pi r} + 2\pi dr - \cancel{2\pi r}}{2\pi r}$$

$$= \frac{dr}{r} \rightarrow (D)$$

= Radial Strain

Hence equating the two value of circumfere-ntial strain given by equation (C) + (D).
we get

$$\frac{dr}{r} = \frac{\sigma_x}{E} + \frac{P_x}{mE} \rightarrow (1)$$

~~On shrinking the two values of circumferential strain given by equations (C) + (D)~~

On shrinking, at the junction there is junction is extension in the inner radius of the outer cylinder and compression in the outer radius of the Inner ~~radius~~. cylinder.

At the junction where $x = r^*$, Increase in the inner radius of outer cylinder.

$$= r^*\left(\frac{\sigma_x}{E} + \frac{P_x}{mE}\right) \longrightarrow (2)$$

But for outer cylinder at the junction, we have

$$\sigma_x = \frac{b_1}{r^{*2}} + a_1 \quad \text{and} \quad P_x = \frac{b_1}{r^{*2}} - a_1$$

where a_1 + b_1 are constants for outer cylinders.

Substitute the value of $\sigma_x + P_x$ in eqn (i), we get Increase in the Inner radius of outer cylinder

$$= r^*\left[\frac{1}{E}\left(\sigma_x + \frac{P_x}{m}\right)\right] = r^*\left[\frac{1}{E}\left(\frac{b_1}{r^{*2}} + a_1\right) + \frac{1}{mE}\left(\frac{b_1}{r^{*2}} - a_1\right)\right]$$

$\|^{ly}$ decrease in the outer radius of the Inner cylinder is obtained from equation (i) as

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$$= -r^* \left[\frac{\sigma_x}{E} + \frac{p_x}{mE} \right] \longrightarrow \text{(iii)}$$

(-ve sign is due to decrease).

But for Inner cylinder at the junction, we have

$$\sigma_x = \frac{b_2}{r^{*2}} + a_2 \quad \& \quad p_x = \frac{b_2}{r^{*2}} - a_2$$

Substituting these values in equation (iii), we get Decrease in the outer radius of inner cylinder.

$$= -r^* \left[\frac{1}{E} \left(\frac{b_2}{r^{*2}} + a_2 \right) + \frac{1}{mE} \left(\frac{b_2}{r^{*2}} - a_2 \right) \right] \rightarrow \text{(iv)}$$

But the original difference in the outer radius of the inner cylinder and Inner radius of the outer cylinder.

= Increase in inner radius of outer cylinder + Decrease in outer radius of the inner cylinder.

$$= r^* \left[\frac{1}{E} \left(\frac{b_1}{r^{*2}} + a_1 \right) + \frac{1}{mE} \left(\frac{b_1}{r^{*2}} - a_1 \right) \right]$$

$$- r^* \left[\frac{1}{E} \left(\frac{b_2}{r^{*2}} + a_2 \right) + \frac{1}{mE} \left(\frac{b_2}{r^{*2}} - a_2 \right) \right]$$

$$= \frac{r^*}{E} \left[\left(\frac{b_1}{r^{*2}} + a_1 \right) - \left(\frac{b_2}{r^{*2}} + a_2 \right) \right] + \frac{r^*}{mE} \left[\cancel{\left(\frac{b_1}{r^{*2}} - a_1 \right) - \left(\frac{b_2}{r^{*2}} - a_2 \right)} \right] \; 0.$$

But from (A) $\frac{b_1}{r^{*2}} - a_1 = \frac{b_2}{r^{*2}} - a_2$.

Hence second part of the above equation is zero.
Hence above equation becomes as original difference of radii at the junction.

$$= \frac{r^*}{E}\left[\left(\frac{b_1}{r^{*2}}+a_1\right)-\left(\frac{b_2}{r^{*2}}+a_2\right)\right]$$

$$= \frac{r^*}{E}\left[\left(\frac{b_1-b_2}{r^{*2}}\right)+(a_1-a_2)\right]$$

$\therefore$ From eqn (B) $\frac{b_1-b_2}{r^{*2}} = a_1-a_2$

$$= \frac{r^*}{E}\left[(a_1-a_2)+(a_1-a_2)\right]$$

$$= \frac{2r^*}{E}(a_1-a_2).$$

The value of a_1 & a_2 are obtained from the given conditions. The value of a_1 is for outer -cylinder & where a_2 is inner cylinder.

A steel cylinder of 300mm external diameter is to be shrunk to another steel cylinder of 150mm internal diameter. After shrinking the diameter at the junction is 250mm ~~internal diameter~~ ~~After shrinking the diameter at the junction is~~ and radial pressure at the common junction is 28 N/mm². Find the original difference in radii at the junction. Take $E = 2\times10^5 N/mm^2$

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Sol^n :-

External dia of outer cylinder = 300 mm.
∴ Radius r_2 = 150 mm
Internal dia of Inner cylinder = 150 mm
∴ Radius r_1 = 75 mm

Diameter at the junction = 250 mm
Radial pressure at the junction $p^* = 28 \, N/mm^2$
Value of $E = 2 \times 10^5 \, N/mm^2$.
using equation (18.3), we get
original difference of radii at the junction

$$= \frac{2r^*}{E}(a_1 - a_2) \rightarrow (1)$$

First, find the value of a_1 & a_2 from the given conditions. These are the constants for outer cylinder and Inner cylinder respectively.

They are obtained by using lame's equations.
For outer cylinder.

$$p_x = \frac{b_1}{x^2} - a_1$$

(i) A junction, $x = r^* = 125$ mm & $p_x = p^* = 28 \, N/mm^2$

(ii) At $x = 150$ mm, $p_x = 0$.

Substitute these two conditions in the above eqn, we get

$$28 = \frac{b_2}{125^2} - a_2 = \frac{b_2}{15625} - a_2 \rightarrow (ii)$$

$$0 = \frac{b_1}{150^2} - a_1 = \frac{b_1}{22500} - a_1 \rightarrow (iii)$$

Solving eqn (ii) & (iii), we get

$b_1 = 1438000$ & $a_1 = 63.6$

For inner cylinder.

(i) at junction, $x = r^* = 125mm$ & $P_x = p^* = 28\ N/mm^2$

(ii) At $x = 75mm$, $P_x = 0$.

$$28 = \frac{b_2}{125^2} - a_2 = \frac{b_2}{15625} - a_2 \quad \text{--- (iv)}$$

$$0 = \frac{b_2}{75^2} - a_2 = \frac{b_2}{5625} - a_2 \quad \text{--- (v)}$$

Solving equation (iv) & (v) we get

$b_2 = -246100$

$a_2 = -43.75$

and Now. Substitute the values of a_2 & a_1 in eqn (i)

we get

Difference of radii at the junction

$$= \frac{2 \times 125}{2 \times 10^5} (63.6 - (-43.75))$$

$$= \frac{125}{10^5} \times 107.35 = \underline{\underline{0.13\ mm}}.$$

<u>Note</u> :- a & b are obtained from boundary conditions which are

(i) at $x = r_1$, $P_x = P_0$ @ the pressure of fluid inside the cylinder &

(ii) at $x = r_2$, $P_x = 0$. @ atmosphere pressure

after the values of a & b the hoop stresses can be calculated at any radius.

UNIT - IV

State of stress in three dimensions

Stress tensor at a point :- [U.Q]

Total stress of any 3D element is determined by the following stress compound

$$\tau_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Stress compound is given by the group of square matrix of stress. The compound of mathematical extity is called stress tensor

where $\sigma_x, \sigma_y, \sigma_z$ = Normal stress

$\tau_{xy}, \tau_{yx}, \tau_{zx}, \tau_{xz}, \tau_{yz}, \tau_{zy}$ = shear stress

Stress invariants :-

A combination of stress at a point donot change with a orientation of co-ordinate axis are called as stress invariants, it denotes as I_1, I_2, I_3

Volumetric strain :-

The ratio of change in volume of the elastic body due to the external force to the original volume.

$$e_v = \frac{\text{change in volume}}{\text{Original volume}}$$

$$\boxed{\therefore e_v = \frac{\delta v}{V}}$$

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Principal plane :- (U.Q)

The plane which passes in such a manner that the resultant stress across them in totally normal stress are known as principal plane.

Principle stress :- (U.Q)

The normal stress across the plane are termed as principal stress.

Problems :-

1. At a point in a stressed body the principal stresses are $100\, MN/m^2$ (T) & $60\, MN/m^2$ (C). Determine the normal stress and the shear stress on a plane inclined at $50°$ to the axis of at major principal stress. Also calculate the maximum shear stress at the point

Given

$\sigma_x = 100\, MN/m^2$ (Tensile),

$\sigma_y = 60\, MN/m^2$ (C) $\Rightarrow -60\, MN/m^2$

$\theta = 50°$

Normal stress, $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \times \cos 2\theta$

$$\sigma_n = \frac{100 + (-60)}{2} + \frac{100 - (-60)}{2} \times \cos 2(50)$$

$$\boxed{\therefore \sigma_n = 6.1\, MN/m^2}$$

Shear stress (τ) $\tau = \frac{\sigma_x - \sigma_y}{2} \times \sin 2\theta$

$$= \frac{100 - (-60)}{2} \times \sin 2(50)$$

$$\boxed{\therefore \tau = 78.78\, MN/m^2}$$

Maximum shear stress $\tau_{max} = \frac{\sigma_x - \sigma_y}{2}$

$$\tau_{max} = \frac{100-(-60)}{2} = 80\ MN/m^2$$

$$\boxed{\therefore \tau_{max} = 80\ MN/m^2}$$

2. At a point in a bracket the stress on two mutually perpendicular plane of are $400\ MN/m^2$ (T) and $300\ MN/m^2$ (C). The shear stress across these planes is $200\ MN/m^2$. Determine magnitude and directions of principal stress and maximum shear stress.

Given

$\sigma_x = 400\ MN/m^2$

$\sigma_y = 300\ MN/m^2$

$\tau_{xy} = 200\ MN/m^2$

Principal stress

$$\sigma = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left[\frac{\sigma_x - \sigma_y}{2}\right]^2 + \tau_{xy}^2}$$

$$= \frac{400+300}{2} \pm \sqrt{\left[\frac{400-300}{2}\right]^2 + (200)^2}$$

$$= 144\ MN/m^2$$

Direction of principal stress

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2\times 200}{400-300}$$

$$\tan 2\theta = \frac{4\cancel{00}}{1\cancel{00}}$$

$$\boxed{\theta = 38^\circ} \text{ (or) } \boxed{\theta = 128^\circ}$$

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Max. shear stress:-

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{556-144}{2} = 206 \, MN/m^2$$

The direction of maximum shear stress with plane

$$\sigma_n = 45^\circ + 38^\circ = 83^\circ$$

Theories of failure:-

The principal theories are

(1) Maximum principal stress theory

(2) Max. principal strain theory

(3) Max. shear stress theory

(4) Total strain energy theory

(5) max. distortion energy theory

Maximum principal stress theory:-

According to this theory failure will occur when the max. principal tensile stress (σ_1) is the complex system reaches the value of the maximum stress at the elastic limit (σ_{et}) in simple tension (or) the minimum principal stress reaches the elastic limit stress (σ_{ec}) in the simple compression

$$\sigma_1 = \sigma_{et} \text{ (tension)}$$

$$\sigma_3 = \sigma_{ec} \text{ (comp.)} = \sigma_3$$

$$\sigma_1 \leq \sigma$$

Approximately correct for ordinary cast Iron and brittle materials.

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Problem:-

1 In a metallic body the principal stresses are $35 MN/m^2$ and $-95 MN/m^2$. The third principal stress being zero. The elastic limit stress in simple tension as well as in simple compression be equal and is $220 MN/m^2$. Find the factor of safety based on the elastic limit if the criterion of failure for the material in the max. principal stress theory.

Given:-

$\sigma_1 = 35 MN/m^2, \sigma_2 = 0, \sigma_3 = -95 MN/m^2$

$\sigma_1 = \sigma t$

$\sigma_1 = \frac{\sigma et}{F.O.S}$ (tension)

$F.O.S = \frac{\sigma et}{\sigma_1} = \frac{220}{35} = 6.28$

$|\sigma_3| = \sigma c$ (compression)

$|\sigma_3| = \frac{\sigma ec}{F.O.S}$

$F.O.S = \frac{\sigma ec}{|\sigma_3|} = \frac{220}{(-95)} = 2.316$

So, the material according to the maximum principal stress theory will fail due to the compressive principal stress.

Factor of safety $= 2.3$

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Maximum principal strain theory:-

This theory associated with st. Venant. The theory states that the failure of a material occurs when the principal tensile strain in material reaches. The strain at the elastic limit in simple tension or when the minimum principal strain reaches the elastic limit in simple compress.

Problem:-

In a steel member, at a point the major principal stress is $180 MN/m^2$ and the minor principal stress is compressive. If the tensile yield point of the steel is $2.25 MN/m^2$. Find the value of the minor principal stress at which yielding will commence, according to each of the following critical of failure.

(1) Max. shear stress

(2) Max. total strain energy and

(3) Max. shear strain energy

Take poission ratio $= 0.26$

Given :-

$\sigma_1 = 180 MN/m^2$,

$\sigma_e = 2.25 MN/m^2$

1. max. shearing stress $\sigma_1 - \sigma_3 = \sigma_e$

$$\sigma_3 = -\sigma_e + \sigma_1$$

$$= 180 + (-225) \Rightarrow -45\,MN/m^2$$

$$\sigma_3 = 45\,MN/m^2 \text{ (comp.)}$$

2. max. total strain energy

$$\sigma_1^2 + \sigma_3^2 - \frac{2}{m}\sigma_1 . \sigma_3 = \sigma_e^2$$

$$180^2 + \sigma_3^2 - 2 \times 0.26 \times 180\,\sigma_3 = 225^2$$

$$\sigma_3^2 - 93.6\,\sigma_3 - 18225 = 0$$

$$\sigma_3 = -96.08\,MN/m^2 \; ; \; \sigma_3 = 96.08\,MN/m^2 \text{ (comp.)}$$

3. max. shear strain energy

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_e^2$$

$$\sigma_1^2 + \sigma_3^2 - \sigma_1 . \sigma_3 = \sigma_e^2$$

$$180^2 + \sigma_3^2 - 180\,\sigma_3 = 225^2$$

$$\sigma_3^2 - 180\,\sigma_3 - 18225 = 0$$

$$\sigma_3 = -72.25\,MN/m^2$$

$$\therefore \sigma_3 = 72.25\,MN/m^2 \text{ (comp.)}$$

Maximum shear stress theory :-

This theory is also called as Guest(or) Trescals theory.

This theory implies that fail will occur when the maximum shear stress τ_{max} in the complex system reaches the value of max. shear stress in simple tension at the elastic limit.

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_{et}}{2}$ [in simple tension]

$\sigma_1 - \sigma_3 = \sigma_{et}$

This theory has been found give quite this theory satisfactory results for ductile materials.

Problem:-

A mild steel shaft 120mm dia. is subjected to a max. torque of 20kN·m and a maximum bending moment of 12 kN·m at a particular section. Find the factor of safety according to the max. shear stress theory if the elastic limit in simple tension is 220 MN/m².

Diameter of the mild steel shaft

$d = 120mm \quad , \quad d = 0.12\,m$

Max. torque $(T) = 20\,kN\text{-}m$

Max. bending moment $(M) = 12\,kN\text{-}m$

$\sigma_{et} = 220\,MN/m^2$

$\sigma_b = \frac{32M}{\pi d^3} = \frac{32 \times 12 \times 10^3}{\pi \times (0.12)^3}$

$\Rightarrow 70.74 \times 10^6\,N/m$

$\Rightarrow 70.74\,MN/m$

$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 20 \times 10^3}{\pi (0.12)^3} \Rightarrow 58.95 \times 10^6\,N/m$

$\Rightarrow 58.95\,MN/m$

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principle stress are given by

$$\sigma = \frac{\sigma_b}{2} \pm \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$= \frac{70.74}{2} \pm \sqrt{\left[\frac{70.74}{2}\right]^2 + 58.95^2}$$

$$\sigma = 35.57 \pm 68.75$$

$$\sigma_1 = 35.57 + 68.75 = 104.12 \, MN/m$$

$$\sigma_2 = 35.57 - 68.75 = -33.38 \, mN/m$$

According to the maximum shear stress theory.

$$\sigma_1 - \sigma_3 = \sigma_t$$

$$\sigma_1 = 104.12 \, mN/m^2 \, ; \quad \sigma_2 = 0 \, ;$$

$$\sigma_3 = -33.38 \, mN/m^2$$

$$\sigma_t = 104.12 - (-33.38) \Rightarrow 137.5 \, MN/m^2$$

$$F.O.S = \frac{\sigma_{et}}{\sigma_t} = \frac{220}{137.5} = 1.6$$

Total strain energy theory :-

This theory which has a thermodynamic analogy and a logic basis is due to high.

This theory states that the failure of a materials occurs when total strain energy theory in material reaches the total strain energy of material at the elastic limit in simple tension.

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Maximum distortion energy theory (or)

shear strain energy theory:-

This theory is also called as. Nises Henky theory.

According to this theory the elastic failurs occure where the stress strain energy per unit volume in the stressed material reaches a value equal to the shear strain energy per unit volume at elastic limit point in simple tension

$$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1) = \sigma_{et}^2$$

The above theory has been to give best results for ductile material for which $\sigma_{et} = \sigma_{ec}$ approximately.

Problem:-

In a material, the principal stresses are $60 MN/m^2$, $48 MN/m^2$ and $-36 MN/m^2$ calculate

(i) Total strain energy

(ii) volumetric strain energy

(iii) shear strain energy

(iv) Factor of safety on the total strain energy ~~crit~~. criterion of the material yields at $12 MN/m^2$

Take $E = 200 MN/m^2$ and $1/m = 0.3$

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(1) Total strain energy per unit volume:

$$= \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2-\left[\frac{2}{m}(\sigma_1\cdot\sigma_2+\sigma_2\cdot\sigma_3+\sigma_3\sigma_1)\right]\right]$$

$$\Rightarrow \frac{1}{2\times200\times10^9}\left[60^2+48^2+(-36)^2-\left[2\times0\cdot3(60\times48+(48\times(-36))+(-36\times60)\right]\right]$$

$$\Rightarrow 1951\ MN/m^3$$

(2) Volumetric strain energy

$$= \frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)^2\left[\frac{1-2/m}{2E}\right]$$

$$= \frac{1}{3}\left[(60+48+(-36)\right]^2\times10^2\left[\frac{1-(2\times0\cdot3)}{2\times200\times10^9}\right]\times10^3$$

$$= 1.728\ kN\cdot m/m^3$$

(3) shear strain energy per unit volume :-

$$= \frac{1}{12C}(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2$$

$$C = \frac{E}{2(1+\frac{1}{m})} = \frac{200\times10^9}{2(1+0\cdot3)} \Rightarrow 76\cdot92\times10^9\ N/m^2$$

$$= \frac{1}{12\times76.923\times10^9}\times10^{-12}\left[(60-48)^2+(48+36)^2+(-36-60)^2\right]$$

$$= 17\cdot78\ kN/m^2$$

(4) Factor of safety

$$\frac{\sigma e^2}{2E} = \frac{(120\times10^6)^2}{2\times200\times10^9}\times10^{-3} \Rightarrow 36\ kN/m^2$$

$$F.O.S = \frac{36}{19.51} \Rightarrow 1.845$$

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UNIT - 5

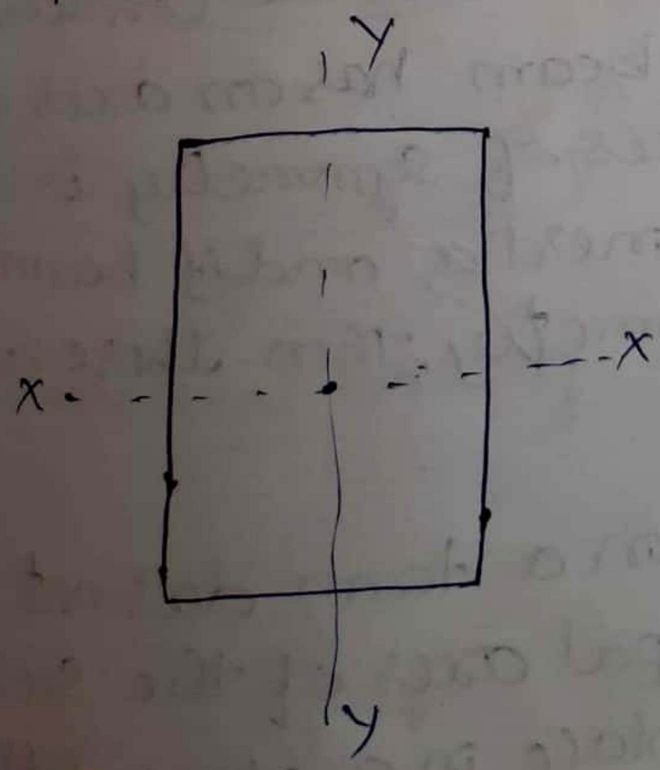
UNSYMMETRICAL BENDING AND SHEAR CENTRE

Unsymmetrical bending of beams of symmetrical and unsymmetrical sections - shear centre - curved beams - Winkler Bach formula - stresses in hooks.

Basic: $= \frac{M}{I} = \frac{f}{y} = \frac{E}{R}$ (Symmetrical Bending)

Important term : product of Inertia

Principal of M.I.



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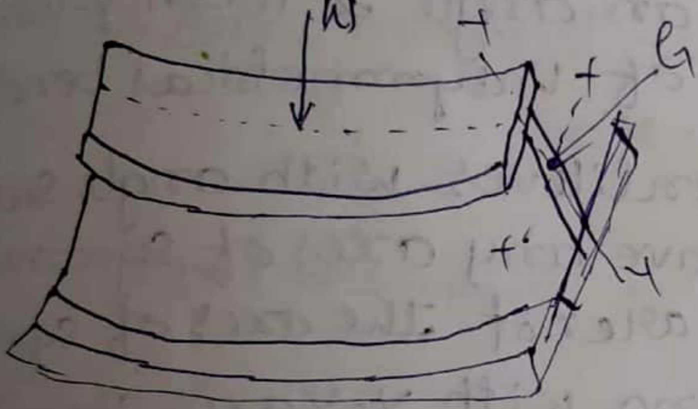
An assumptions is taken that section of the beam is symmetrical about the plane of bending. This condition is satisfied if the plane of the loads contains the axis of symmetry of all the sections of the beam. Beam sections like circular, rectangular, square and I sections are symmetrical about ~~about~~ the plane of bending and about the neutral axis, while T- section and channel section with web horizo-ntal are symmetrical about the plane of bending but unsymmetrical about neutral axis. Then a beam section such as as angle-section, is not symmetrical about both the centroidal axis. If the c/s of the beam has an axis of symmetry then this axis of symmetry is always a principal axis of inertia, and if beam section has two axes of symmetry. then these are two principal axes.

If the load line on a beam does not coincide with one of the principal axes of the section. the bending takes place in a plane different from the plane of principal axes. this type of bending is known as unsymmetrical bending.

there are two reasons for Unsymmetrical bending as follows.

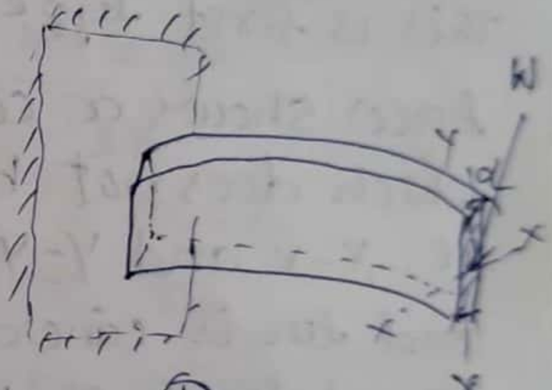
1. The section is symmetrical like I section, rectangular section, circular section but the load line is inclined to both the principal axes.

2. the section itself is unsymmetrical like angle section or channel section with vertical web and load line is along any centroidal axis.



ⓐ

symmetrical bending



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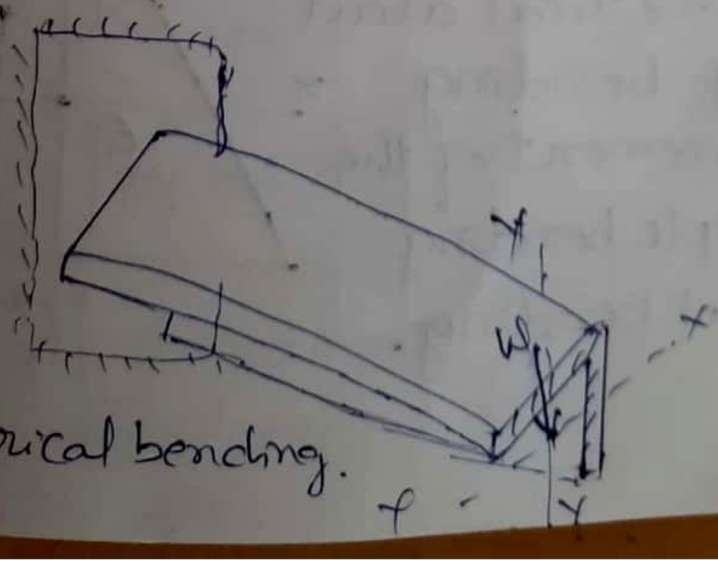
Unsymmetrical bending
Symmetrical section
but oblique load.



Unsymmetrical bending.

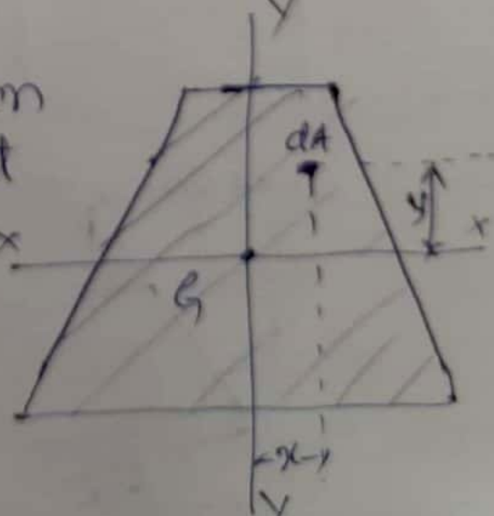
fig(a) shows a beam with I section with load line co-inciding with y-y principal axis. I-section has two axes of symmetry and both these axes are principal axis. section is symmetrical about y-y plane, i.e the plane of bending. This type of bending is known as symmetrical bending.

fig(b) shows a cantilever with rectangular section, which has two axes of symmetry which are principal axes but the load line is inclined at an angle α with y-y axis. This is first type of unsymmetrical bending.

fig(c) shows a cantilever with angle section which does not have any axes of symmetry i.e., x-x and y-y are not the axes of symmetry load line is coinciding with y-y axis. This is second type of unsymmetrical bending.

Principal Axes :-

fig shows a beam section which is symmetrical about the plane of bending y-y. is requirement of the theory of simple bending (or) symmetrical bending.



G is the centroid of the section. XX and YY are the two perpendicular axes passing through the centroid. Say the bending moment on the section (in the plane YY of the beam) is M about the axis XX. Consider a small element of area $\underline{dA}$ with (x, y) co-ordinates.

Stress on the element $f = \frac{M}{I_{xx}} \cdot y$.

Force on the element, $dF = \frac{My\,dA}{I_{xx}}$.

Bending moment about Y-Y axis,

$$dM = \frac{My\,x\,dA}{I_{xx}}$$

Total moment, $M' = \int \frac{My\,x\,dA}{I_{xx}}$

If no bending is to take place about YY axis, then

$$M' = 0$$

$$\int \frac{Myx\,dA}{I_{xy}} = 0$$

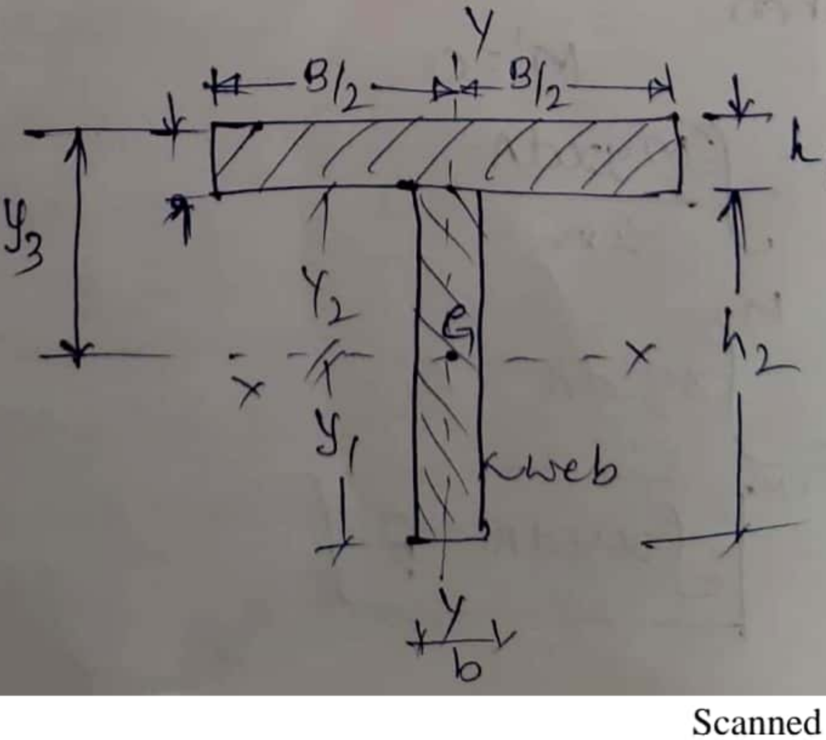
$$\frac{M}{I_{xx}} \int xy \cdot dA = 0$$

$$\boxed{\int xy\,dA = 0}$$

The expression $\int xy\,dA$ is called a product of Inertia, of the area about X-X and Y-Y axis, represented by I_{xy}. If the product of inertia is zero about the two co-ordinates axes passing through the centroid, then the bending is symmetrical or Pure bending. Such axes are called Principal moments of Inertia.

The Product of Inertia may be +ve, negative or zero depending upon the Section and co-ordinate axes. The product of Inertia of a section with respect to two perpendicular axes is zero, if either one of the axes is an axis of Symmetry.

Example :- Show that product of inertia of a T-section about a centroidal axis is zero.



T-section with flange $B \times h_1$ and web $b \times h_2$. The section is symmetrical about YY axis. Say G is the centroid the section on the axis YY, and XX & YY are the centroidal axes.

$$I_{xy} = I_{xy}' \text{ for flange} + I_{xy}'' \text{ for web}$$

For flange x varies from

$$-B/2 \text{ to } +B/2$$

For web x varies from

$$-b/2 \text{ to } +b/2$$

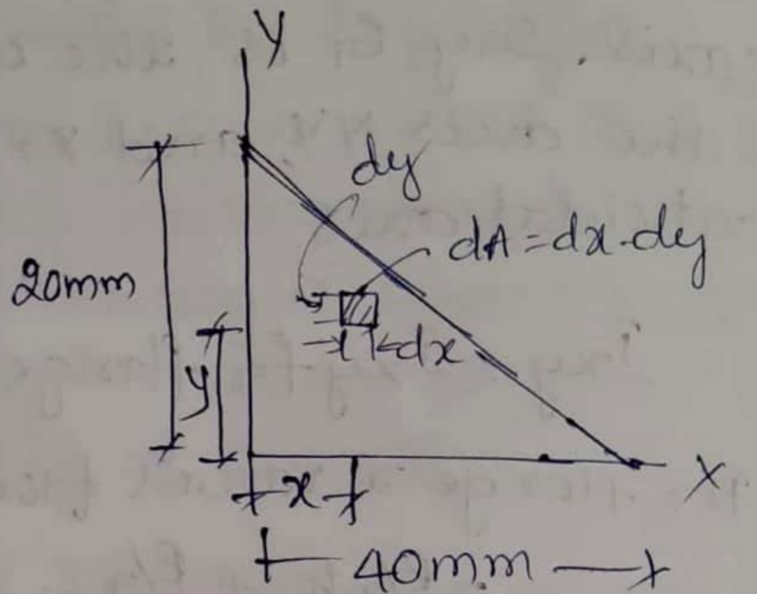
$$\text{Now, } I_{xy} = \int_{y_2}^{y_3} \int_{-B/2}^{+B/2} xy\,dx\,dy + \int_{-y_1}^{y_2} \int_{-b/2}^{+b/2} xy\,dy\,dy$$

$$= \int_{y_2}^{y_3} \left| \frac{x^2}{2} \right|_{-B/2}^{+B/2} y\,dy + \int_{-y_1}^{y_2} \left| \frac{x^2}{2} \right|_{-b/2}^{+b/2} y\,dy$$

$$= 0 \times \int_{y_2}^{y_3} y\,dy + 0 \times \int_{-y_1}^{y_2} y\,dy = 0$$

(for flange) (for web)

Determine the Product of inertia about axes x & y for a triangular Section shown in fig.



Consider a small element of area dA at co-ordinates x, y

Product of inertia about x-y axis.

$$I_{xy} = \int_0^{20} \int_0^{2y} xy\,dx\,dy$$

Note that limiting value of x = 40 mm
= 2 × limiting value of y

$$= \int_0^{20} \left[\int_0^{2y} x\,dx \right] y\,dy \text{ and also } = \int_0^{40} \left[\int_0^{x/2} y\,dy \right] x\,dx$$

$$= \int_0^{20} \left| \frac{x^2}{2} \right|_0^{2y} y\,dy = \int_0^{20} 2y^2 \cdot y\,dy = \int_0^{20} 2y^3\,dy$$

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$$= \left| \frac{y^4}{2} \right|_0^{20} = \frac{20^4}{2} = 80000 mm^4$$

Determination of Principal axes

Fig shows a section with centroid G, XX and YY are two co-ordinates axes passing through G. Say UU and VV is another set of axes passing through the centroidal G and inclined at an angle θ to the x-y co-ordinate. Consider an element of area dA at point p having co-ordinates (x, y). Say u, v are co-ordinates of the point p in U-V co-ordinate axes.

So, $u = GA' = GD + DA' = GD = AE$

where $GD = GA\cos\theta = x\cos\theta$

$AE = DA' = y\sin\theta$

$\Rightarrow u = x\cos\theta + y\sin\theta$

$v = GB' = PA' = PE - A'E$

$v = PE - AD$ since $A'E = AD$

$v = PA\cos\theta - x\sin\theta = y\cos\theta - x\sin\theta$

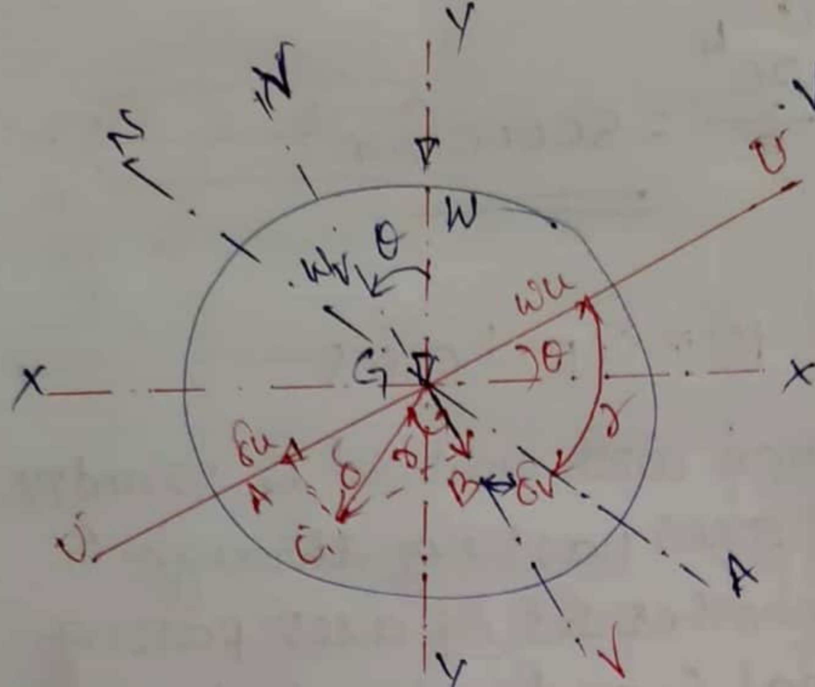
$|||^{ly}$ x, y co-ordinates can be written in terms of u, v co-ordinates

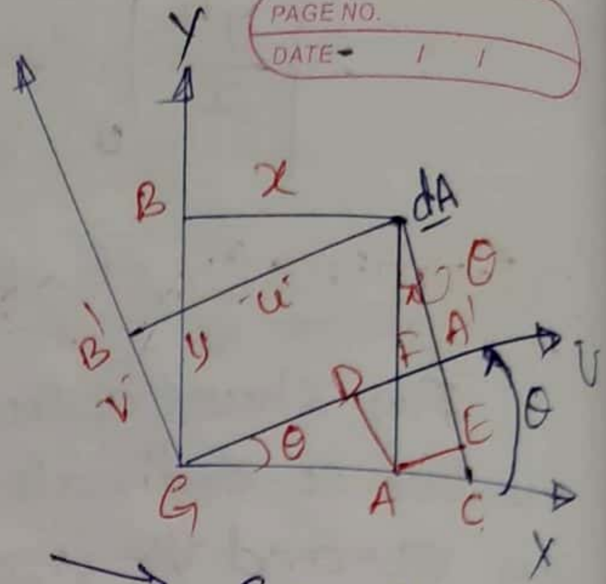
$x = GC - AC = GC - A'F = U\cos\theta - V\sin\theta$

(as $PA' = V$ and $GA' = U$)

$y = GB = PA = AF + FP = A'C + FP =$

$$\boxed{y = u\sin\theta + v\cos\theta.}$$





$GA = \delta u$
$GB = \delta v$
$GC = \delta$
NA = Neutral Axis

Second moment of area about U-U

$$I_{uu} = \int v^2 dA = \int (y\cos\theta - x\sin\theta)^2 dA$$

$$= \int y^2\cos^2\theta\, dA + \int x^2\sin^2\theta\, dA - \int 2xy\sin\theta\cos\theta\, dA$$

$$= I_{xx}\cos^2\theta + I_{yy}\sin^2\theta - \int \sin 2\theta . xy\, dA$$

$$= I_{xx}\cos^2\theta + I_{yy}\sin^2\theta - \int \sin 2\theta . xy\, dA$$

$$= I_{xx}\cos^2\theta + I_{yy}\sin^2\theta - I_{xy}\sin 2\theta$$

$$= \frac{1}{2}(I_{xx} + I_{yy}) + \frac{1}{2}\left[I_{xx} - I_{yy}\right]\cos 2\theta - I_{xy}\sin 2\theta \longrightarrow (1)$$

Second moment of area about V-V

$$I_{vv} = \int u^2 dA = \int (x\cos\theta + y\sin\theta)^2 dA$$

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$$= \int x^2 \cos^2\theta \, dA + \int y^2 \sin^2\theta \, dA + \int 2xy \sin\theta\cos\theta \, dA$$

$$= I_{yy}\cos^2\theta + I_{xx}\sin^2\theta + I_{xy}\sin 2\theta$$

$$= \frac{1}{2}(I_{xx}+I_{yy}) + \frac{1}{2}(I_{yy}-I_{xx})\cos 2\theta$$

$$+ I_{xy}\sin 2\theta \longrightarrow (2)$$

From equations (1) and (2)

$$I_{uu}+I_{vv} = I_{xx}(\sin^2\theta+\cos^2\theta) + I_{yy}(\sin^2\theta+\cos^2\theta)$$

$$= I_{xx} + I_{yy} \longrightarrow (3)$$

Product of Inertia about UV axes

$$I_{uv} = \int uv\,dA = \int (x\cos\theta + y\sin\theta)(y\cos\theta - x\sin\theta)\,dA$$

$$= \int xy(\cos^2\theta - \sin^2\theta)\,dA + \int y^2 \sin\theta\cos\theta\,dA$$

$$- \int x^2 \sin\theta\cos\theta\,dA$$

$$I_{uv} = I_{xy}\cos 2\theta + I_{xx}\frac{\sin 2\theta}{2} - I_{yy}\frac{\sin 2\theta}{2}$$

But as per the condition of pure bending (4) symmetrical bending $I_{uv}=0$, Then U & V will be the principal axes.

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(or) $2 I_{xy} \cos 2\theta + (I_{xx} - I_{yy}) \sin 2\theta = 0$

$$\tan 2\theta = \frac{2 I_{xy}}{I_{yy} - I_{xx}} = \frac{I_{xy}}{(I_{yy} - I_{xx})/2} \rightarrow (4)$$

say θ_1 & θ_2 are two values of θ given by eqn (4)

$$\boxed{\theta_2 = \theta_1 + 90^\circ}$$

$$\sin 2\theta_1 = \frac{I_{xy}}{\sqrt{\left(\frac{I_{yy} - I_{xx}}{2}\right)^2 + I_{xy}^2}} \text{ and}$$

$$\cos 2\theta_1 = \frac{(I_{yy} - I_{xx})/2}{\sqrt{\left(\frac{I_{yy} - I_{xx}}{2}\right)^2 + I_{xy}^2}}$$

Substituting these value of $\sin 2\theta_1$ & $\cos 2\theta_1$

$$(I_{uu})_{\theta_1} = (I_{xx} + I_{yy}) + \frac{\frac{1}{2}(I_{xx} - I_{yy}) \cdot \frac{1}{2}(I_{yy} - I_{xx})}{\sqrt{\left[\frac{1}{2}(I_{yy} - I_{xx})\right]^2 + I_{xy}^2}}$$

$$- \frac{I_{xy} \cdot I_{xy}}{\sqrt{\left[\frac{1}{2}(I_{yy} - I_{xx})\right]^2 + I_{xy}^2}}$$

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$$(I_{uu})_{\theta_1} = \tfrac{1}{2}(I_{xx}+I_{yy}) - \sqrt{\tfrac{1}{2}(I_{yy}-I_{xx})^2 + I_{xy}^2} \longrightarrow (5)$$

lly

$$(I_{vv})_{\theta_1} = \tfrac{1}{2}(I_{xx}+I_{yy}) + \sqrt{[\tfrac{1}{2}(I_{yy}-I_{xx})]^2 + I_{xy}^2} \quad (6)$$

Now for $\boxed{\theta_2 = \theta_1 + \pi/2}$

$$\sin 2\theta_2 = \sin(2\theta_1+\pi) = -\sin 2\theta_1$$

$$\cos 2\theta_2 = \cos(2\theta_1+\pi) = -\cos 2\theta_1$$

substituting these values in eqn (1) & (2)

$$(I_{uu})_{\theta_2} = \tfrac{1}{2}(I_{xx}+I_{yy}) + \sqrt{[\tfrac{1}{2}(I_{yy}-I_{xx})]^2 + I_{xy}^2} \Rightarrow (7)$$

$$(I_{vv})_{\theta_2} = \tfrac{1}{2}(I_{xx}+I_{yy}) - \sqrt{[\tfrac{1}{2}(I_{yy}-I_{xx})]^2 + I_{xy}^2} \rightarrow (8)$$

From equation 5, 6, 7, 8 we learn that

$$(I_{uu})_{\theta_1} = (I_{vv})_{\theta_2} \ \& \ (I_{vv})_{\theta_1} = (I_{uu})_{\theta_2}$$

Maximum and minimum values of I_{uu} & I_{vv}

$$I_{uu} = \frac{1}{2}(I_{xx}+I_{yy}) + \frac{1}{2}(I_{yy}-I_{xx})\cos 2\theta + I_{xy}\sin 2\theta$$

For maximum value of I_{uu},

$$\frac{dI_{uu}}{d\theta} = 0$$

ie $\frac{1}{2}(I_{yy}-I_{xx})(-2\sin 2\theta) + I_{xy} \times 2\cos 2\theta = 0$

$$\boxed{\tan 2\theta = \frac{I_{xy}}{(I_{yy}-I_{xx})/2}}$$

This shows that the values of $(I_{uu})_{\theta_1}$ and $(I_{vv})_{\theta_1}$ are the maximum and minimum values of I_{uu} and I_{vv}. These value are called the principal values of moment of inertia as $I_{uv}=0$. The directions θ_1 and θ_2 are called the principal directions.

Moment of Inertia about any axis

If the principal moments of Inertia I_{uu} and I_{vv} are known then moment of inertia about any axis inclined at an angle θ to the principal axes can be determined. Say u, v are the co-ordinates of an element of area dA in the U-V principal axes system. x & y are the co-ordinates axes inclined at an angle θ to the U-V axes.

x co-ordinate of element $= u\cos\theta - v\sin\theta$

y co-ordinate of element $= u\sin\theta + v\cos\theta$

Moment of Inertia $I_{yy} = \int x^2 dA = \int (u\cos\theta - v\sin\theta)^2 dA$

$= \int u^2\cos^2\theta\, dA + \int v^2\sin^2\theta\, dA - \int 2uv\sin\theta\cos\theta\, dA$

$= I_{vv}\cos^2\theta + I_{uu}\sin^2\theta - 0$ since $\int uv\,dA = 0$

$= I_{vv}\cos^2\theta + I_{uu}\sin^2\theta$

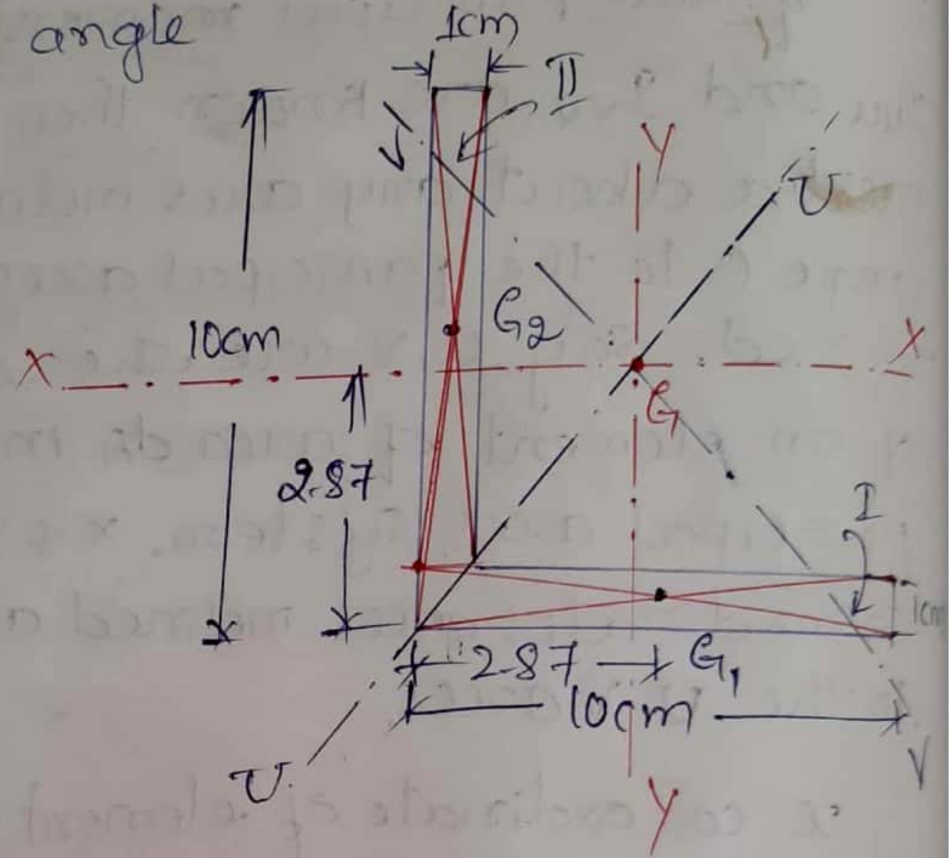
||ly $I_{xx} = I_{uu}\cos^2\theta + I_{vv}\sin^2\theta$

From equations (9) & (10)

$$I_{xx} + I_{yy} = I_{uu} + I_{vv} = J.$$

Polar moment of inertia about an axis passing through G and normal to the sections.

① Determine the principal moments of Inertia for the equal angle



Let us consider the angle section in Two portions I & II as shown and determine the Position of the centroid.

$$\bar{x} = \bar{y} = \frac{10 \times 0.5 + 9 \times 5.5}{19} = 2.87\,cm$$

(due to symmetry $\bar{x} = \bar{y}$)

moment of Inertia $I_{xx} = I_{yy}$

$$= \frac{10 \times 1^3}{12} + 10(2.87 - 0.5)^2 + \frac{9 \times 1^3}{12} + 9(2.87 - 0.5)^2$$

$$= 0.833 + 56.169 + 0.750 + 50.552$$

$$= \underline{108.304\,cm^4}$$

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co-ordinates of centroid of portion I
$= [(5-2.87), -(2.87-0.5)] = (2.13, -2.37)$

co-ordinates of portion - II
$= \{-(2.87-0.5), (5.5-2.87)\} = (-2.37, 2.63)$

Product of Inertia

$I_{xy} = 10(2.13)(-2.37) + 9(2.63)(-2.37)$,

(as the product of Inertia about their own centroidal axes is zero, since portion I & II are rectangles).

So $I_{xy} = -50.481 - 56.098 = \underline{-106.579} \text{ cm}^4$

If θ = angle of the principal axes UU with respect to X-axis.

$$\tan 2\theta = \frac{I_{yy}}{(I_{yy} - I_{xx})/2} = \frac{106.579}{0.0} = \infty$$

$$2\theta = 90^\circ \Rightarrow \theta = 45^\circ$$

Principal angles are $\theta_1 = 45^\circ$, $\theta_2 = 90^\circ + 45^\circ = \underline{135^\circ}$

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Principal moments of Inertia:-

$$I_{uu} = \frac{1}{2}(I_{xx}+I_{yy}) + \frac{1}{2}(I_{xx}-I_{yy})\cos 2\theta_1 \mp I_{xy}\sin 2\theta_1$$

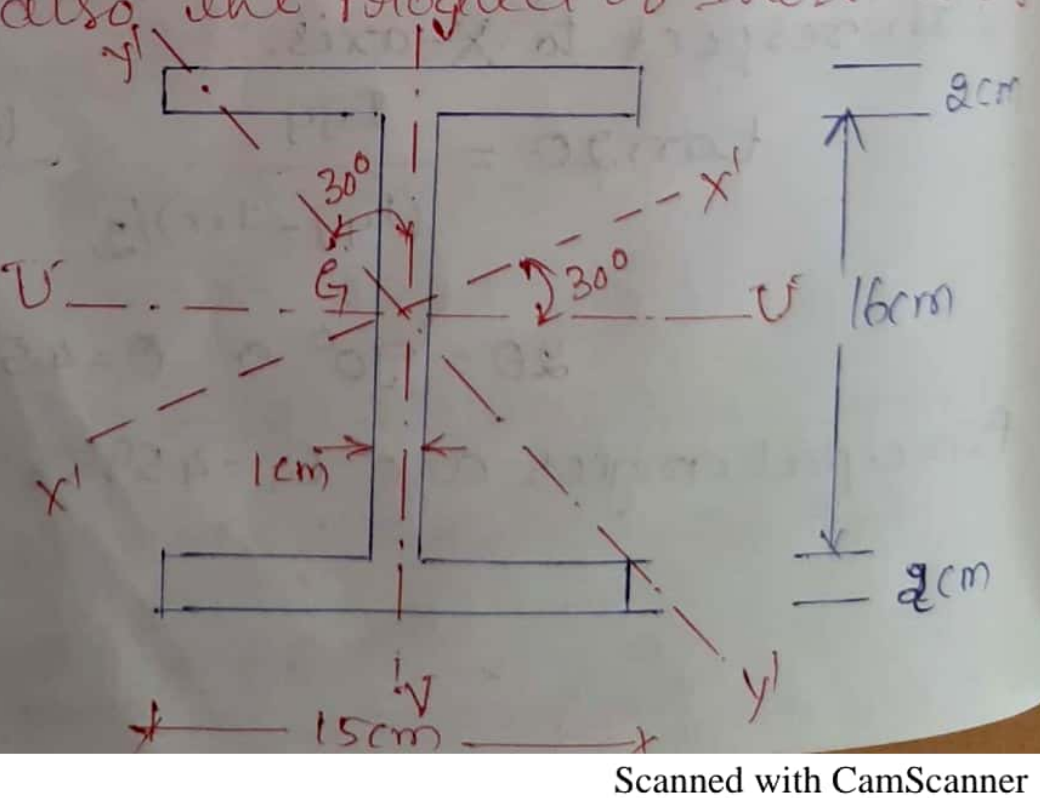
$$= \frac{1}{2}(108.304 + 108.304) + \frac{1}{2}\times 0 \times \cos 90^\circ$$
$$+ 106.579 \sin 90^\circ$$

$$= 108.304 + 106.579 = \underline{\underline{214.883\,cm^4}}$$

$$I_{vv} = I_{xx} + I_{yy} - I_{uu} = 2\times 108.304 - 214.883$$

$$\boxed{I_{vv} = 1.725\,cm^4}$$

(2) Fig shows a in I section 15cm × 20cm. Axes x'x' and y'y' are inclined at an angle of 30° to the axes of symmetry. Determine the moment of Inertia about these axes. Calculate also the Product of Inertia $I_{x'y'}$



The I Section shown has two ~~axes~~ of symmetry i.e, UU and VV ~~passing~~ passing ~~are~~ through the Centroid G. therefore, these are the principal axes and I_{uu} & I_{vv} are the principal moments of Inertia. the angle of inclinations of UU and VV axes with respect x'x' and y'y' axes is $\theta = 30°$.

$\sin^2\theta = 0.25 \quad \cos^2\theta = 0.75$

$$I_{y'y'} = I_{vv}\cos^2\theta + I_{uu}\sin^2\theta$$

$$I_{x'x'} = I_{uu}\cos^2\theta + I_{vv}\sin^2\theta$$

$$I_{uu} = \frac{15\times20^3}{12} - \frac{14\times16^3}{12} = 10,000 - 4778.667$$

$$= \underline{5221.33\ cm^4}$$

$$I_{vv} = \frac{2\times15^3}{12} + \frac{16\times1^3}{12} + \frac{2\times15^3}{12}$$

$$= 562.5 + 1.333 + 562.5$$

$$\boxed{I_{vv} = 1126.33\ cm^4}$$

$$I_{y'y'} = 1126.333\times0.75 + 5221.33\times0.25$$

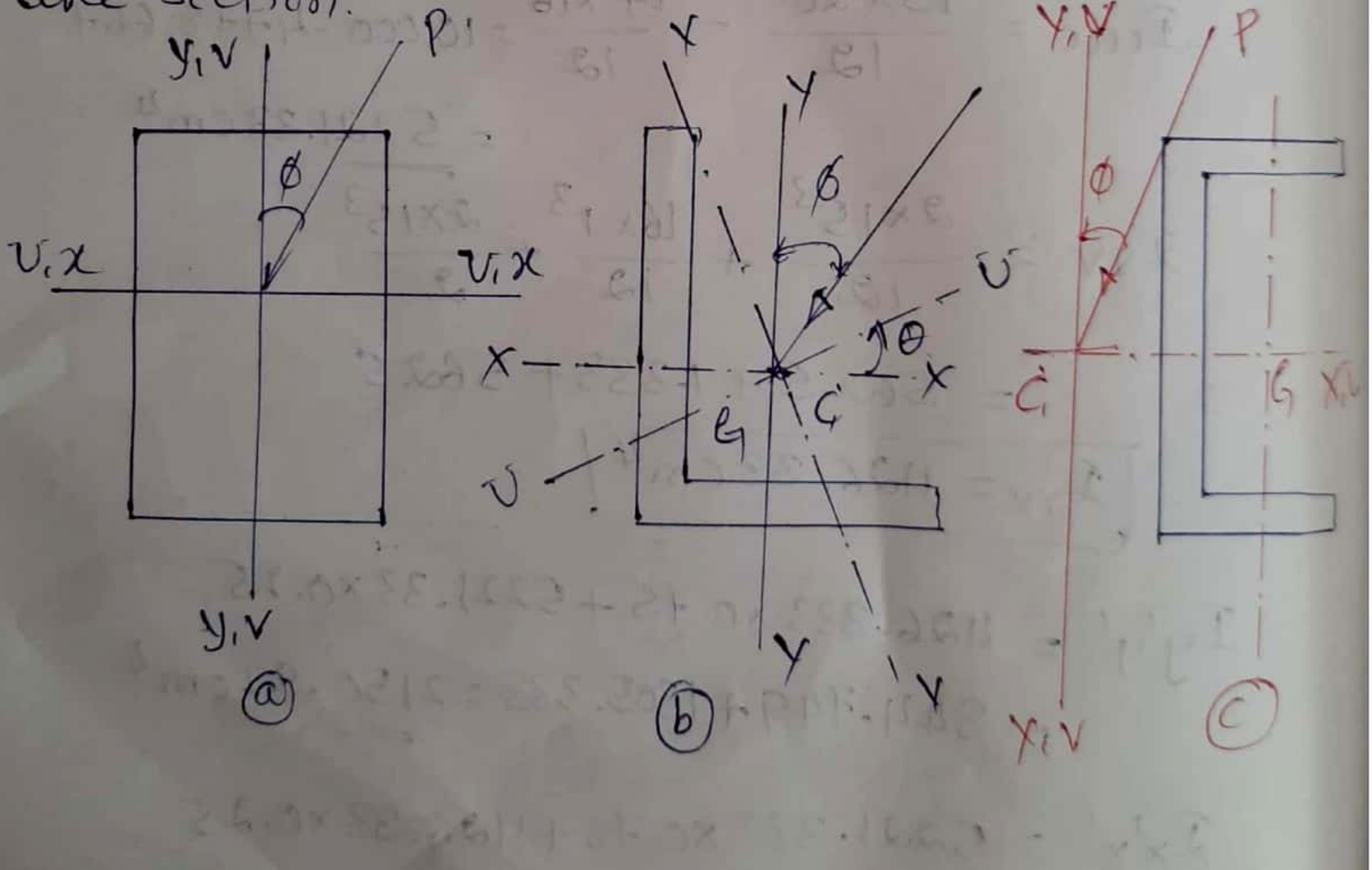
$$= 844.749 + 1305.333 = \underline{2150.82\ cm^4}$$

$$I_{x'x'} = 5221.333\times0.75 + 1126.33\times0.25$$

$$= 3915.999 + 281.583 = \underline{4197.582\ cm^4}$$

Stresses due to Unsymmetrical Bending

When the load line on a beams does not coincide with one of the principal axes of the section, unsymmetrical bending takes place. Fig (a) shows a rectangular section symmetrical about XX & YY axis or with U-U and V-V principal axes. Load line is inclined at an angle ϕ to the principal axes VV, and passing through G (Centroid) or C (Shear Centre) the section.



Fig(b) shows an angle section which does not have any axis of symmetry. Principal axes UU and V'V' are inclined to axes XX and YY at an angle θ. load line is inclined at an angle ϕ to the vertical or at an angle $(90 - \phi - \theta)$ to the axis U-U. load line is passing through G (centroid of the section) or C (shear centre)

Fig(c) shows a channel section which has one axis of ~~of~~ symmetry i.e., XX. Therefore, UU and VV are the principal axis. G is the centroid of the section while C is the shear centre. load line is inclined at an angle ϕ to the vertical (or the axis VV) & passing through the shear centre of the section.

Shear centre for any transverse section of a beam is the point of intersection of the bending axis and the plane of transverse section. If a load passes through the shear centre there will be only bending of the beam and no twisting will occur. If a section has two axes of symmetry, then shear centre coincides with the centre of gravity or centroid of the section as in the case of a rectangular, circular or I-section.

For sections having one axis of Symmetry only, shear centre does not coincide with centroid but lies on the axis of symmetry as shown in the case of a channel section.

For a beam subjected to symmetrical bending only, following Assumptions are made:

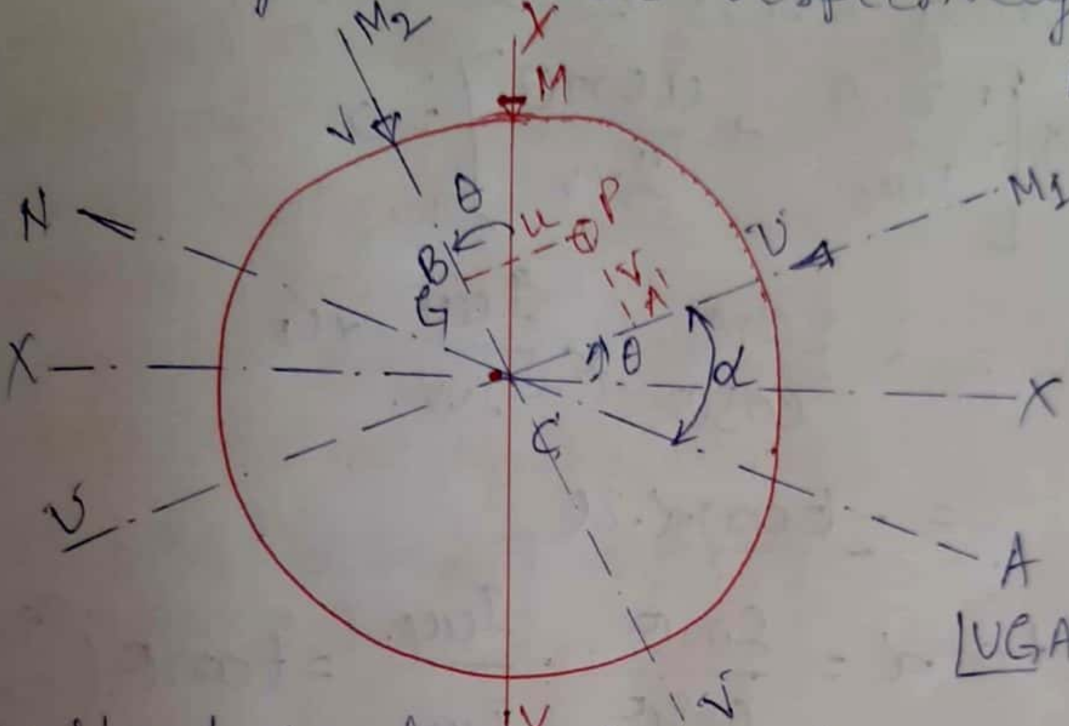
1. The beam is initially straight and uniform section through out
2. load or loads are assumed to act through the axis of bending.
3. load or loads act in a direction Perpendicular to the bending axes and load line passes through the centre of transverse section.

Fig shows a cross section of a beam subjected to bending moment M, in the plane YY. G is the centroid of the section and XX and YY are two co-ordinate axis passing through G. moreover UU and VV are the principal axes inclined at an angle θ to the XX and YY axis respectively. Let us determine the stresses due to bending at the point p having the co-ordinates u, v corresponding to principal axes. moment in the plane YY can be resolved into two components. M_1 & M_2

M_1, moment in the plane U-U = $M \sin\theta$

M_2, moment in the plane VV = $M \cos\theta$ ①

The components M_1 & M_2 have their axes along VV and U.U respectively



$M_1 = M\sin\theta$

$M_2 = M\cos\theta$

$GA' = U$

$GB = V$

$\angle UGA = \alpha$

Resultant bending stress at point P.

$$f_b = \frac{M_1 \cdot u}{I_{VV}} + \frac{M_2 \cdot V}{I_{uu}} = \frac{M\sin\theta}{I_{VV}} + \frac{M\cos\theta \cdot v}{I_{uu}}$$

$$= M\left[\frac{v\cos\theta}{I_{uu}} + \frac{u\sin\theta}{I_{VV}}\right] \longrightarrow ①$$

The exact nature (whether tensile or comp) depends upon the quadrant in which the point P lies. In other words sign of co-ordinates u & v is to be taken into account while determining the resultant bending stress.

The equation of the neutral axis can be determined by considering the resultant bending stress. At the neutral axis bending stress is zero i.e.,

$$M\left[\frac{v\cos\theta}{I_{uu}} + \frac{u\sin\theta}{I_{vv}}\right] = 0$$

$$v = -\frac{\sin\theta}{\cos\theta} \times \frac{I_{uu}}{I_{vv}} \times u$$

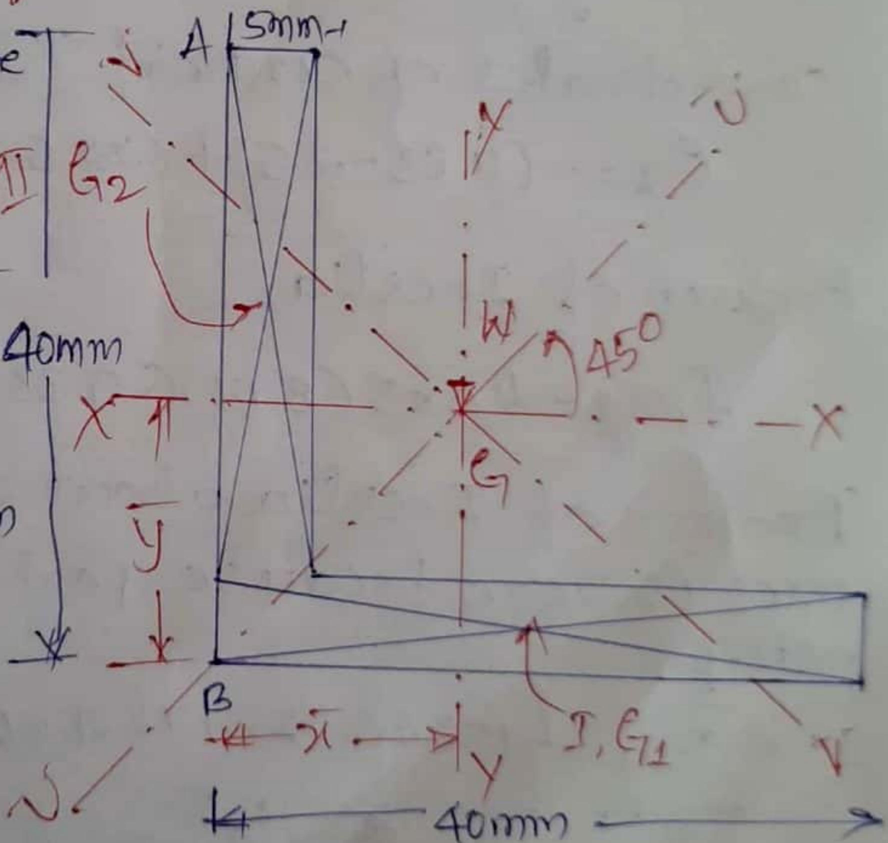
$$= -\tan\alpha \cdot u$$

where $\tan\alpha = \frac{\sin\theta}{\cos\theta} \cdot \frac{I_{uu}}{I_{vv}} = \tan\theta\left(\frac{I_{uu}}{I_{vv}}\right)$

This is the equation of a straight line passing through the centroid G of the section. All the points of the section on one side of the neutral axis have stress of the same nature and all the points of the section on the other side of the neutral axis have stresses of opposite nature.

① A 40mm × 40mm × 5mm angle section shown in the fig is used as a simply supported beam over a span of 2.4 metres. It carries a 0.20kN of load along the line YG. where G is the centroid of the section. Determine resultant bending stresses on point A, B & C i.e., outer corners of the section, along the middle section of the beam

Let us first determine the position of the centroid



$$\bar{x} = \bar{y} = \frac{40\times5\times2.5 + 35\times5\times22.5}{200+175}$$

$$= \frac{500 + 3937.5}{375} = 11.83\text{mm}$$

Moment of Inertia

$$I_{xx} = \frac{5\times35^3}{12} + 5\times35(22.5-11.83)^2 + \frac{40\times5^3}{12} + 40\times5(11.83-2.5)^2$$

$= 17864.583 + 19923.537 + 416.667 + 17409.780$

$= 55614.537 \, mm^4 = \underline{\underline{5.561 \times 10^4 \, mm^4}}$

$= \underline{\underline{I_{yy}}}$ (because it is equal angle section)

Co-ordinates of G_1 (centroid of portion I)

$= +(20 - 11.83), -(11.83 - 2.5)$

$= \underline{\underline{(8.17, -9.33)}}$

co-ordinates of centroid

$G_2 = -(11.83 - 2.5) + (22.5 - 11.83) = -9.33, +10.67$

Product of Inertia,

$I_{xy} = 40 \times 5 (8.17)(-9.33) + 35 \times 5 (-9.33)(10.67)$

(Product of Inertia about their centroidal axes is zero because portion I & II are regular strip).

$I_{xy} = -15245.22 - 17421.44 = -32666.66 \, mm^4$

$$\boxed{I_{xy} = -3.266 \times 10^4 \, mm^4}$$

<u>Principal angle, θ</u>

$$\tan 2\theta = \frac{I_{xy}}{\frac{1}{2}(I_{yy} - I_{xx})} = \frac{-3.266 \times 10^4}{0} = \infty$$

$= \tan 90^\circ \therefore \theta = 45^\circ$

Principal Moment of Inertia

$I_{uu} = \frac{1}{2}(I_{xx}+I_{yy}) + \frac{1}{2}(I_{xx}-I_{yy})\cos 90^0 - I_{xy}\sin 90^0$

$= \frac{1}{2}(5.561+5.50)\times 10^4 + \frac{1}{2}\times 0\times \cos 90^0 + 3.266\times 10^4$

$= (5.561 + 3.266)\times 10^4 = 8.827\times 10^4 \text{ mm}^4$

$I_{vv} = I_{xx} + I_{yy} - I_{uu} = (5.561 + 5.561 - 8.827)\times 10^4$

$\boxed{I_{vv} = 2.295\times 10^4 \text{ mm}^4}$

Bending moment $M = \frac{WL}{4} = \frac{0.20\times 10^3 \times 2.4\times 10^3}{4}$

$\boxed{M = 0.120\times 10^6 \text{ N-mm}}$

component of bending moment.

$M_1 = M\sin 45^0 = 0.120\times 0.707\times 10^6 = 84.84\times 10^3 \text{ N-mm}$

$M_2 = M\cos 45^0 = 0.120\times 0.707\times 10^6 = 84.84\times 10^3 \text{ N-mm}$

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U-V co-ordinates of the points

Point A, $x = -11.83$, $y = 40 - 11.83 = 28.17$

$$u = x\cos\theta + y\sin\theta = -11.83 \times 0.707 + 28.17 \times 0.707$$
$$= \underline{11.55\,mm}$$

$$v = y\cos\theta - x\sin\theta = 28.17 \times 0.707 + 11.83 \times 707$$
$$= 28.28\,mm.$$

Point B. $x = -11.83$, $y = 11.83$

$$u = 28.17 \times \cos 45^\circ - 11.83 \sin 45^\circ$$
$$= 28.17 \times 0.707 - 11.83 \times 0.707 = \underline{\underline{11.55\,mm}}$$

$$v = -11.83 \times 0.707 - 28.17 \times 0.707 = \underline{\underline{-28.28\,mm}}$$

Resultant bending stresses at points A, B & C.

$$f_A = \frac{M_1 u}{I_{VV}} + \frac{M_2 v}{I_{VV}} = 84.84 \times 10^3 \left[\frac{11.55}{2.295 \times 10^4} + \frac{28.28}{8.827 \times 10^4}\right]$$

$$\boxed{f_A = 69.88\,N/mm^2}$$

$$f_B = 84.84 \times 10^3 \left[\frac{-11.627}{2.295 \times 10^4} + \frac{0}{8.827 \times 10^4}\right] = -42.98\,N/mm^2$$

$$f_C = 84.84 \times 10^2 \left[\frac{11.55}{2.295 \times 10^4} - \frac{28.28}{8.827 \times 10^4}\right] = \underline{\underline{+15.51\,N/mm^2}}$$

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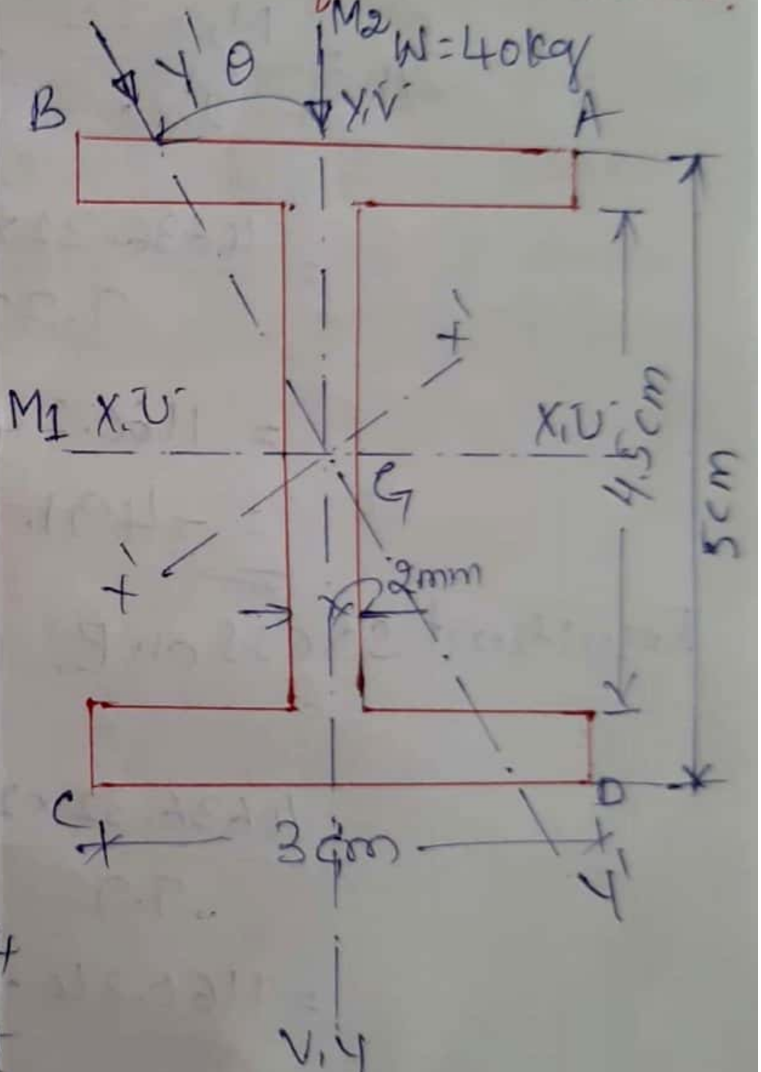
Q Fig shows I section of a cantilever 1.2m long subjected to a load W = 40kg at free end along the direction Y'G inclined at 15° to the vertical. Determine the resultant bending stress at corners A & B. at the fixed section of the cantilever.

Soln I section is symmetrical about XX and YY axis, therefore XX & YY are the principal axes UU & VV.



moment of Inertia

$$I_{uu} = I_{xx} = \frac{3\times5^3}{12} - \frac{2.8\times4.5^3}{12}$$

$$= 31.25 - 21.26$$

$$= 9.99\ cm^4$$

$$I_{vv} = I_{yy} = \frac{0.25\times2\times3^3}{12} + \frac{4.5\times(0.2)^3}{12}$$

$$= 1.125 + 0.003 = 1.128\ cm^4$$

Maxm. Bending moment

$M = WL = 40\times120 = 4800$ kg-cm

$M_1 = M\sin15° = 4800\times0.2588$

$= 1242.24$ kg-cm

$M_2 = M\cos15° = 4800\times0.9659$

$= 4636.32$ kg-cm

Due to bending moment M_1, there will be tensile stresses at points B and C and compressive stresses at points D and A.

Due to bending moment M_2 there will be tensile stress on points A & B and compressive stress on points C & D.

Resultant bending stress on A,

$$f_A = \frac{M_2 \times 2.5}{I_{xx}} - \frac{M_1 \times 1.5}{I_{yy}}$$

$$= \frac{4636.32 \times 2.5}{9.99} - \frac{1242.24 \times 1.5}{1.128}$$

$$= 1160.24 - 1651.91$$

$$= -491.67 \text{ kg/cm}^2$$

Resultant stress on B, $f_B = \frac{M_2 \times 2.5}{I_{xx}} + \frac{M_1 \times 1.5}{I_{yy}}$

$$= \frac{4636.32 \times 2.5}{9.9} + \frac{1242.24 \times 1.5}{1.128}$$

$$= 1160.24 + 1651.91 = 2812.15 \text{ kg/cm}^2$$

Deflection of Beams due to Unsymmetrical Bending.

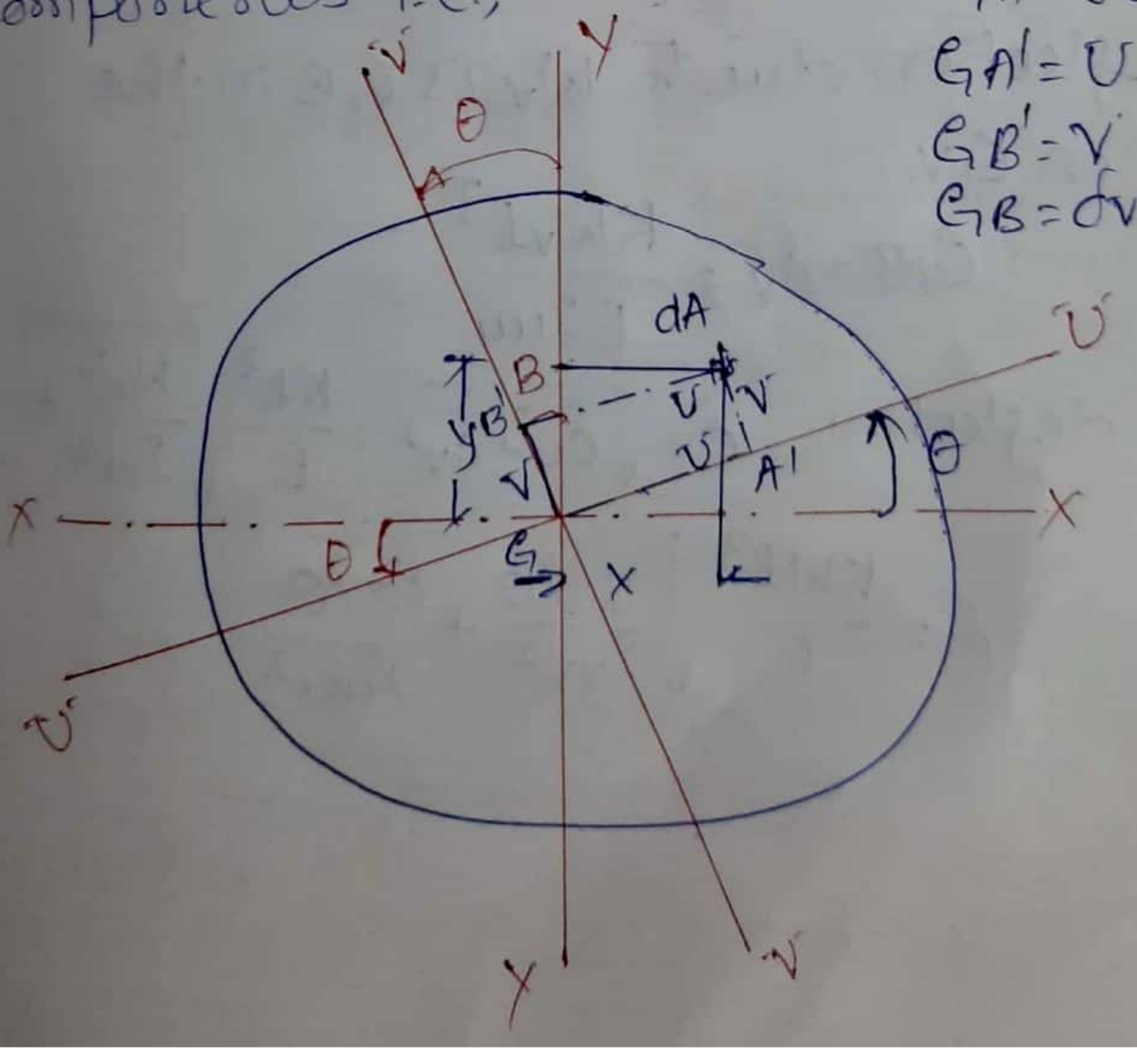
Fig shows the transverse section of a beam with centroid G. X-X and Y-Y are two rectangular co-ordinate axes and U-U and V-V are the principal axes inclined at an angle θ to the XY set of co-ordinate axes. Say the beam is subjected to a load W along the line YG. This load can be resolved into two components i.e.,

$GA = \delta U$

$GA' = U$

$GB' = V$

$GB = \delta V$



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$W_u = W \sin\theta$

(along UG direction)

$W_v = W \cos\theta$

(along VG direction)

Say deflection due to W_u is GA in the direction GU.

i.e. $GA = \delta_u = \frac{K \cdot W_u \cdot l^3}{E I_{vv}}$

where 'K' is a constant depending upon the end conditions of the beam & position of the load along the beam.

Deflection due to W_v is GB in the direction GV.

$$GB = \delta_v = \frac{K W_v l^3}{E I_{uu}}$$

Total deflection, $\delta = \sqrt{\delta_u^2 + \delta_v^2} = \frac{K l^3}{E} \sqrt{\frac{W_u^2}{I_{vv}^2} + \frac{W_v^2}{I_{uu}^2}}$

$$\delta = \frac{K W l^3}{E} \sqrt{\frac{\sin^2\theta}{I_{vv}^2} + \frac{\cos^2\theta}{I_{uu}^2}}$$

Total deflection δ is along the direction GC at the angle γ to VV axis

$$\tan\gamma = \frac{CG}{GB} = \frac{GA}{GB} = \frac{W_u}{I_{VV}} \times \frac{I_{uu}}{W_V}$$

$$= \frac{W\sin\theta}{W\cos\theta} \times \frac{I_{uu}}{I_{VV}} = \tan\theta \frac{I_{uu}}{I_{VV}}$$

comparing this with the second moment of area about V-V axis.

$$\cancel{\tan\gamma = \frac{CG}{GB} = \frac{GA}{GB}} =$$

$$\boxed{\tan\alpha = \tan\theta \times \frac{I_{uu}}{I_{VV}}}$$

where α is the angle of inclination of the neutral axis with respect to UU axis

and $\boxed{\tan\gamma = \tan\theta \cdot \frac{I_{uu}}{I_{VV}}}$

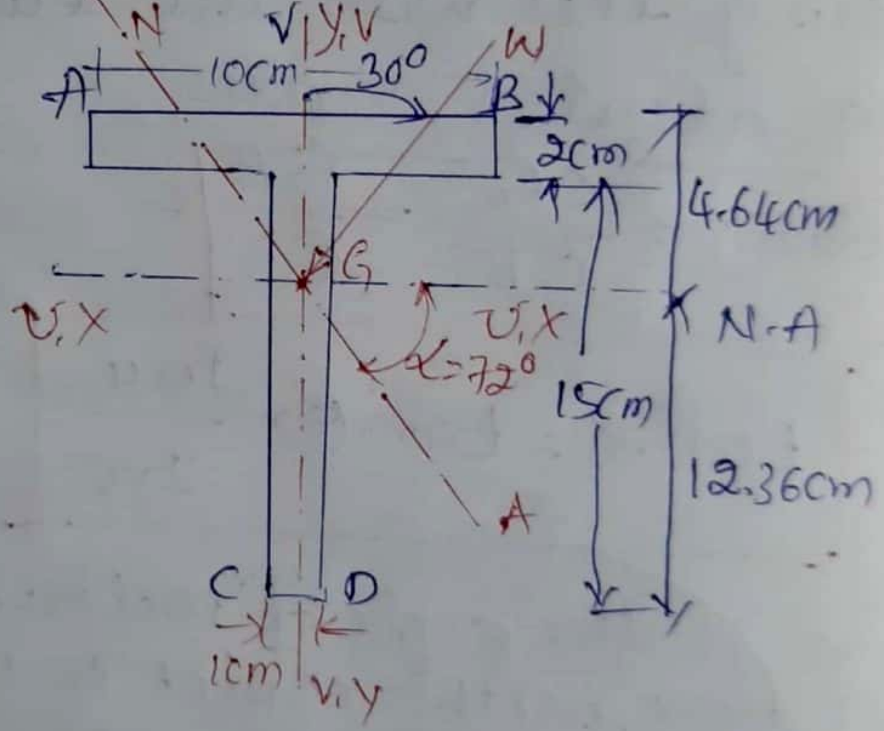
where γ is the angle of inclination of direction of δ with respect to VV axis

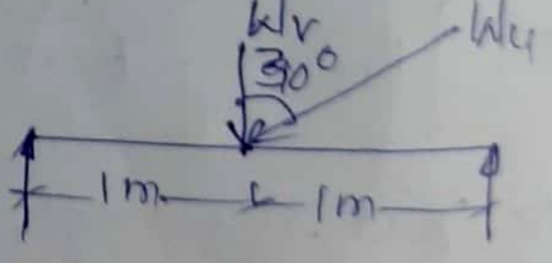
$\gamma = \alpha$, showing thereby that resultant deflection δ takes place in a direction perpendicular to the neutral axis.

① A simply supported beam of length 2 metres carries a central load 4 kN inclined at 30° to the vertical and passing through the centroid of the section. Determine

① maximum tensile stress ② maximum compressive stress ③ deflection due to the load. ④ direction of neutral axis. Given $E = 200 \times 10^5\ N/cm^2$

Soln.





let us first determine the position of the centroid of the T-section shown in the fig.

$$\bar{y} = \frac{15\times1\times7.5+10\times2\times(15+1)}{15+20} = 12.36cm$$

The section is symmetrical about vertical axis. therefore the principal axes pass through the centroid G and are along U-U and V-V axes shown.

$$S_0 = I_{xx} = I_{uu} = \frac{10\times12^3}{12} + 20(4.64-10)^2 + \frac{1\times15^3}{12}$$
$$+15(12.36-7.5)^2$$

$= 6.667 + 264.992 + 281.250 + 354.294$

$\boxed{I_{xx} = 907.203cm^4}$

$$I_{yy} = I_{vv} = \frac{2\times10^3}{12} + \frac{15\times1^3}{12} = 166.667 + 1.250$$
$$= 167.917cm$$

load, $W = 4000N$

components of W, $W_v = 4000\times\cos30^\circ = 4000\times0.866$
$= 3464N$

$W_u = 4000\times\sin30^\circ = 4000\times0.50 = 2000N$

Bending moment, $M_v = \frac{W_v\times l}{4} = \frac{3464\times200}{4} = 173,200N\text{-}cm$

Bending moment, $M_u = \frac{W_u\times l}{4} = \frac{2000\times200}{4} = 100,000N\text{-}cm$

Due to M_v there will be maximum compressive stress on A & B and maximum tensile stress at C & D

Due to M_u there will be maximum compressive stress at B and D and maximum tensile stress at A & D.

So maximum compressive stress at B.

$$f_B = \frac{M_v \times 4.64}{I_{uu}} + \frac{M_u \times 5}{I_{vv}}$$

$$= \frac{173200 \times 4.64}{907.203} + \frac{100000 \times 5}{167.917} = 885.852 + 2977.661$$

$$= 3863.5\ N/cm^2 = \underline{\underline{38.63\ N/mm^2}}$$

Maximum tensile stress at C

$$f_c = \frac{M_v \times 12.36}{I_{uu}} + \frac{M_u \times 0.5}{I_{vv}}$$

$$= \frac{173200 \times 12.36}{907.203} + \frac{100000 \times 0.5}{167.917}$$

$$= 2359.727 + 297.776 = \underline{\underline{2657.493\ N/cm^2}}$$

$$\boxed{f_c = 26.57\ N/mm^2}$$

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Deflection $\delta = \frac{KWL^3}{E} \sqrt{\frac{\sin^2\theta}{I_{vv}^2} + \frac{\cos^2\theta}{I_{uu}^2}}$

$K = 1/48$ as the beam is simply supported and carries a concentrated load at its centre

$$\delta = \frac{KWl^3}{E I_{uu}} \sqrt{\sin^2\theta \times \left(\frac{I_{uu}}{I_{vv}}\right)^2 + \cos^2\theta}$$

Now $\sin\theta = 0.5 \therefore \sin^2\theta = 0.25$

$\cos\theta = 0.866, \cos^2\theta = 0.75$

$$\delta = 1/48 \times \frac{4000 \times (200)^3}{200 \times 10^5 \times 907.203} \sqrt{0.25 \times \left(\frac{907.203}{169.917}\right) + 0.75}$$

$$= 0.0367 \sqrt{0.25 \times 28.50 + 0.75}$$

$$\delta = 0.0367 \times 2.8065 = 0.103\text{ cm} = 1.03\text{ mm}$$

Position of the neutral axis :-

$$\tan\alpha = \tan\theta \frac{I_{uu}}{I_{vv}} = \tan 30^\circ \times \frac{907.203}{169.917}$$

$$= 0.5774 \times 5.339 = 3.0828$$

$$\boxed{\alpha = 72^\circ}$$

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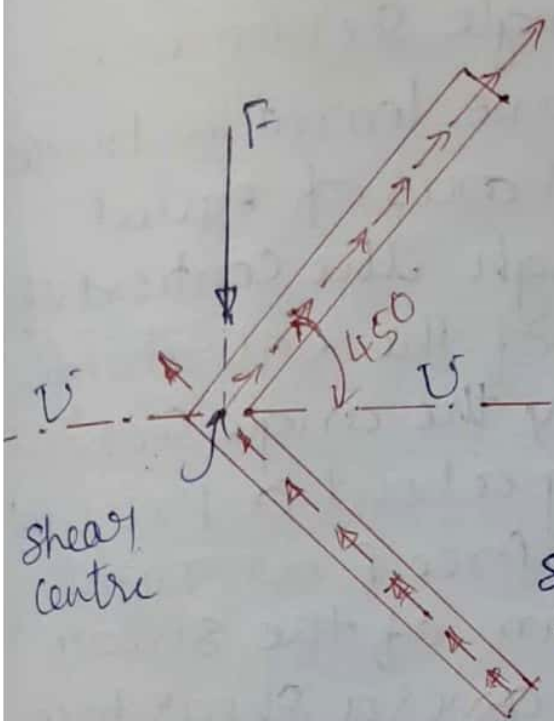
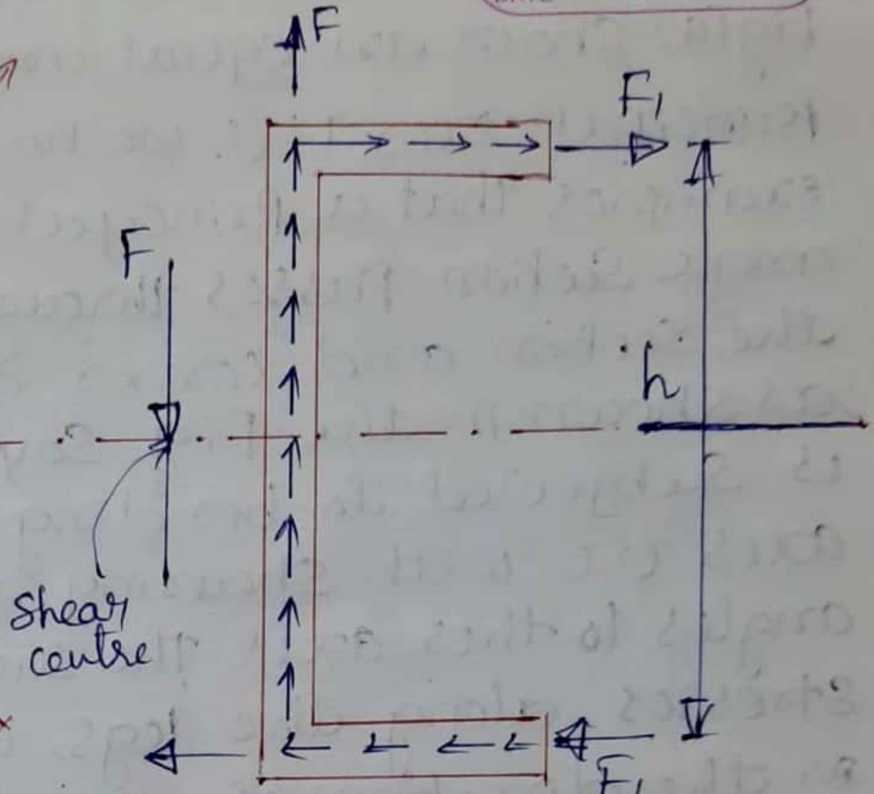
Shear centre.

The distribution of shear stresses in the transverse section of a beam subjected to bending moment M and shear force F. Summation of shear stresses over the section of the beam gives a set of forces which must be in equilibrium with the applied shear force F. In case of symmetrical sections such as rectangular and I-sections, the applied shear is balanced by the set of shear forces summed over the rectangular section or over the flanges and web of I section and the shear centre coincides with the centroid of the section. If the applied load is not placed at the shear centre, the section twists about this point and this point is also known as point about which the applied shear force is balanced by the set of shear forces obtained by summing the shear stresses over the section.

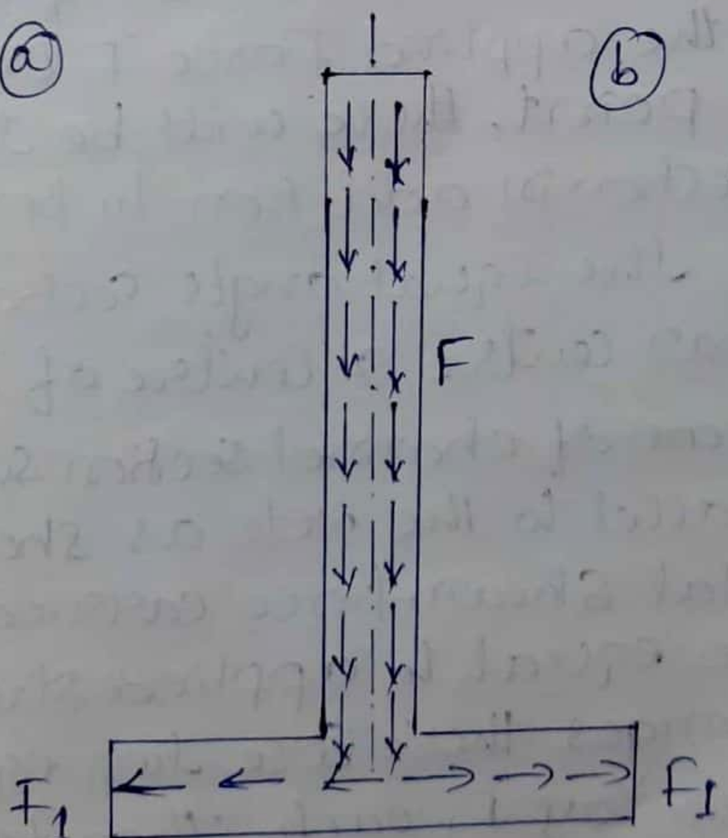
For unsymmetrical sections such as angle section and channel section, summation of shear stresses in each leg gives a set of forces which should be in equilibrium with the applied shear force.



(a)



(b)



Shear centre (c)

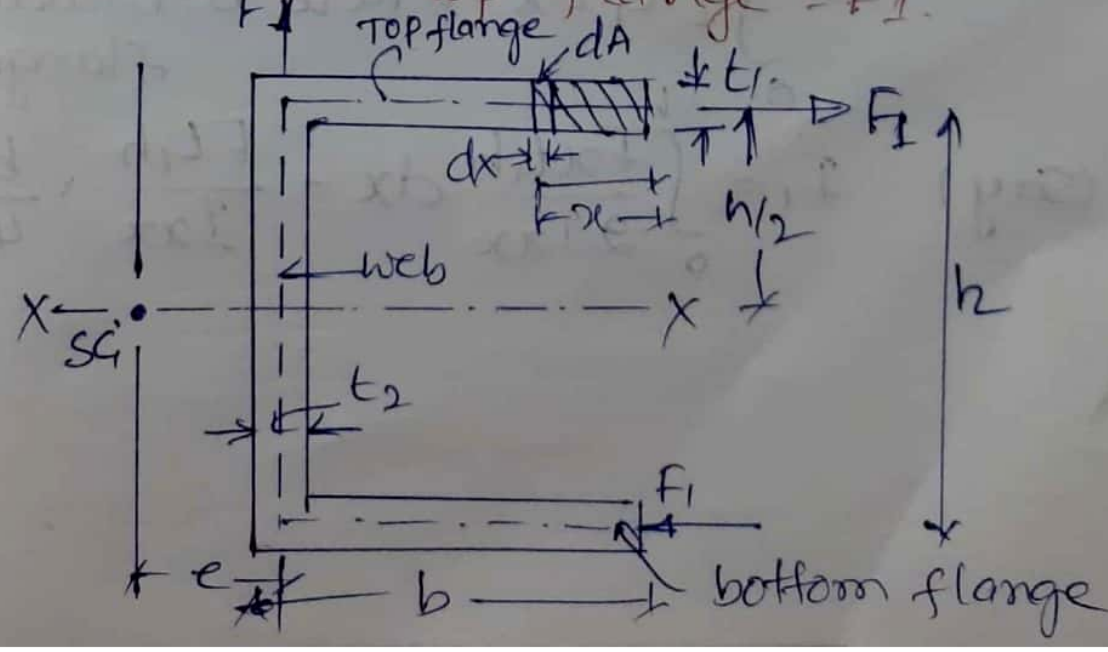
Fig(a) shows an equal angle section with principal axes U.V. we have learnt in previous examples that a principal axis of equal angle section passes through the centroid of the section and corner of the equal angle as shown in the Fig. Say the angle section is subjected to bending about a principal axis UV with shearing force F at right angles to this axis. The sum of the shear stresses along the legs, gives a shear force in the direction of the corner of the angle and unless the applied force F is applied through this point, there will be twisting of the angle section in addition to bending. This point of the equal angle section is called its shear centre or centre of twist.

For a beam of channel section subjected to loads parallel to the web, as shown in fig (b). The total shear force carried by the web must be equal to applied shear force F, then in flanges there are two equal & opposite forces say F_1 each. Then for equilibrium $F \times e$ is equal to $F_1 \cdot h$

and we can determine the position of the shear centre along the axis of symmetry ie $e = \frac{F_1 \times h}{F}$

$||^{ly}$ Fig (c) shows a T-section and its centre. vertical force in web F is equal to the applied shear force F and horizontal force F_1 in two portions of the flange balance each other at shear centre.

① Fig shows a channel channel section with flanges $b \times t_1$ and web $h \times t_2$, X-X in the horizontal symmetric axis of the section. Say F is the applied shear force, vertically downwards. Then shear force in the web will be F upwards. Say the shear force in the top flange $= F_1$.



Shear stress in the flange at a distance of x from right hand edge.

$$= \frac{Fa\bar{y}}{I_{xx} \times t}$$

where F = applied shear force

$a\bar{y} = (t_1, x)\, h/2$, first moment of area about axis $X-X$.

$t = t_1$ (thickness of the flange)

$$q = \frac{F.t_1 x}{I_{xx}.t_1} \times h/2 = \frac{Fxh}{2I_{xx}}$$

Shear force in elementary area $(t_1 dx = dA) = q.dA = q.t_1.dx$

Top shear force in top flange

$$= \int_0^b q.t_1.dx$$ where b = breadth of flange

(Say) $$F_1 = \int_0^b \frac{Fxt_1h}{2I_{xx}}dx = \frac{Ft_1h}{I_{xx}} \times \frac{b^2}{4}$$

There will be equal and opposite shear force in the bottom flange.

Say shear centre is at a distance of 'e' from web along the symmetric axis XX.

Then for equilibrium.

$$F \cdot e = F_1 h = \frac{F t_1 h^2 b^2}{4 I_{xx}} \quad \text{or} \quad e = \frac{t_1 b^2 h^2}{4 I_{xx}}$$

Moment of Inertia, $I_{xx} = \frac{t_2 h^3}{12} + \frac{2 \times b \times t_1^3}{12} + 2 \times b \times t_1 (h/2)^2$

in which, the expression $\frac{2 b t_1^3}{12}$ is negligible in comparison to other terms.

$$I_{xx} = \frac{t_2 h^3}{12} + \frac{b t_1 h^2}{2} = \frac{h^2}{12}(t_2 h + 6 b t_1)$$

If we take $b t_1$ = area of flange = A_f

$h t_2$ = area of web = A_w

Then
$$\boxed{e = \frac{3 b A_f}{A_w + 6 A_f} = \frac{3b}{6 + \frac{A_w}{A_f}}}$$